

# Optimal Fiscal Policy in Overlapping Generations Models\*

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## Abstract

In this paper, we explore the proposition that the optimal capital income is zero using an overlapping generations model. We prove that for a large class of preferences, the optimal capital income tax along the transition path and in steady state is non-zero. We do find a special case that results in a zero capital income tax in steady state. However, if either the government can condition taxes on age or generations live two periods, then the uniform commodity taxation principle is a sufficient condition to ensure zero optimal capital taxes. We simulate the model to confirm the connection between commodity taxation and capital income taxation. We find that the optimal commodity tax over the life-cycle is J-shaped. When taxes cannot be conditioned on age, this policy can be imperfectly replicated by setting a non-zero capital income tax.

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# 1 Introduction

This paper explores the proposition that the optimal capital income tax is zero. The standard view is that capital returns should not be taxed at all. This view is built on a well-established theory of optimal fiscal policy. In standard neoclassical growth models with infinitely-lived consumers, Judd (1985) and Chamley (1986) show that the optimal policy predicts zero capital taxes in the long run.<sup>1</sup> However, this result cannot be extended in general to a life-cycle model. In a quantitative paper, Escolano's (1992) uses a large scale overlapping generations model and finds that positive capital income taxes can be optimal. He also shows that the welfare gains associated to implement the optimal policy are small. More recently, Erosa and Gervais (2002) study the same problem in an environment where the government uses age-specific taxes. They find that if the government can condition taxes on age, the zero capital income tax results of the infinite-lived consumer model can be extended to life-cycle economies under a broad class of preferences.

The present paper studies the optimal fiscal policy problem in an overlapping generations model where the government **cannot** condition taxes on age. The objective is to generalize and to extend the work of Escolano (1992) and Erosa and Gervais (2002), as well as tie the findings to the commodity tax literature. The main theoretical result we find is that the optimal capital income tax is in general different from zero, both in the transition path and in the long run. We provide a sufficient condition for zero capital taxation, and show that it is more restrictive than the standard uniform commodity taxation principle required when taxes can be conditioned on age. For a general class of preferences commonly used to evaluate fiscal policies, the optimal capital income tax is different from zero. However, we do find a special class of preferences that results in a zero capital income tax in steady state. These results provide a theoretical foundation of Escolano's (1992) quantitative findings, and are complementary to the results derived by Erosa and Gervais (2002).

To provide some intuition for the non-zero capital income result it is useful to relate the present findings with the economic intuition presented in Judd (1999) for an infinitely-lived consumer economy. We know from general equilibrium theory that the static Arrow-Debreu model can be applied to a dynamic context by indexing commodities by time, so does the principles from static commodity tax literature. As a result, a positive capital income tax is equivalent to a commodity tax on the time  $t$  good that

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<sup>1</sup>Several papers have extended this result to more general class of economies that include endogenous growth (Jones, Manuelli and Rossi (1997)), aggregate shocks (Chari, Christiano and Kehoe (1994)), and open economies (Razin and Sandka (1995)) and find similar results, capital returns should not be taxed at all.

grows exponentially in  $t$ . In an infinitely-lived consumer economy, if preferences are separable and exhibit some degree of substitutability, this policy entails an ever-growing distortion between the marginal rate of substitution and the marginal rate of transformation. Given that individuals have a preference to smooth consumption, they prefer a constant consumption tax to an ever-increasing consumption tax. This policy can be implemented by removing the tax rate on capital income and replacing it with a tax on labor income. In contrast, if individuals live a finite number of periods the distortions associated with this policy are less important because for a given generation current consumption and period  $T$  consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes.

In an overlapping generations model consumption and hours worked are not constant over the life-cycle even in the steady state. As a result, the government has an incentive to tax consumption when it is relatively higher. When taxes cannot be conditioned on age, this policy can be imperfectly mimicked by setting a non-zero capital tax. Nevertheless, if we interpret consumption at different ages like different goods that can be taxed at a different rate, in general, we find that the optimal policy implies an increasing consumption tax over the life-cycle, except on the early stages where the optimal consumption tax decreases.

The key to the general result is the existence of consumers of different ages making the same type of decisions (consumption/savings and supplying labor) at a given point in time. Restricting the tax policy to age-independent taxes imposes additional constraints to the government problem that leads to non-zero capital income taxes. However, along the transition path and in the long run, the uniform commodity tax result is a sufficient condition to ensure zero optimal capital income taxes, if either the government can condition taxes on age, or generations live two periods and the old does not supply labor.

To confirm the connection between commodity taxation and capital income taxation we simulate the model for an empirically reasonable intertemporal utility function and a robust demographic structure. The quantitative findings are consistent with Escolano (1992). If the government cannot condition taxes on age, given some plausible choices of parameter values, the optimal policy can be consistent with the observed tax rates.<sup>2</sup> Nevertheless, the optimal capital tax predicted by the model changes when the government can condition taxes on age, as predicted by Erosa and Gervais

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<sup>2</sup>Mendoza, Tesar and Razin (1994) document that most OECD economies have effective capital income taxes that are different from zero. In particular, for the US the average capital income tax over the period 1965-95 is around 35%, and in the U.K. and Germany it is around 37% and 23.5% respectively.

(2002). For preferences that satisfy the uniform commodity tax result the optimal capital tax across ages is zero. This zero capital income tax also implies an equivalent commodity tax of zero as in the infinite-lived consumer model. With preferences that are not consistent with the uniform commodity tax result, the optimal capital income tax subsidizes the asset returns of the younger generations and taxes the asset return of the older ones. For this class of preferences the equivalent commodity tax over the life-cycle is J-shaped. The capital income subsidy is equivalent to a decreasing consumption tax in the early stages of the life-cycle, that reverts later on when the capital income tax is positive. Hence, in the next stages of the life-cycle the consumption tax is increasing. This pattern confirms the basic intuition that consumption is taxed when is relatively high. When taxes cannot be conditioned on age, this non-constant consumption tax can be imperfectly replicated by setting a non-zero capital tax that modifies the relative Arrow-Debreu price for consumption goods.

The paper is organized as follows. Section 2 describes the behavior of market economy, and section 3 defines the government problem and derives the sufficient conditions for the zero capital tax result. Section 4 further characterizes the optimal policy and illustrates the basic results using numerical simulations. Finally, section 5 concludes.

## 2 The economy

The economy is an overlapping generations model with production and two goods, a consumption-capital good and labor. Agents live  $I \geq 2$  periods and each cohort is populated by a continuum of identical households, represented by the interval  $[0, 1]$ . Without loss of generality, the population is assumed to be stationary and its total size is constant.<sup>3</sup>

There is a representative firm that produces aggregate output  $Y_t$  using a constant returns to scale production function  $F(K_t, L_t)$ , using aggregate capital  $K_t$  and aggregate labor  $L_t$  as primary inputs. Labor is measured in efficiency units. The production function  $F : R_+^2 \rightarrow R_+$  is strictly concave, monotone, continuously differentiable. Capital depreciates each period at a constant rate  $\delta \in (0, 1)$  and there is no exogenous technological change. These assumptions imply that in competitive factor markets firms will make zero profits, hence it is unnecessary to specify firms' ownership. Then, each period prices are determined by

$$r_t = F_{Kt} - \delta, \quad \forall t, \tag{1}$$

$$w_t = F_{Lt}, \quad \forall t, \tag{2}$$

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<sup>3</sup>This is not an important assumption for the basic results and simplifies notation.

where  $r_t$  denotes the interest rate net of depreciation and  $w_t$  is the wage rate per efficiency unit of labor. Let  $C_t$  and  $L_t$  denote aggregate consumption and labor respectively as

$$C_t = \prod_{i=1}^I c_t^i, \quad \forall t, \quad (3)$$

$$L_t = \prod_{i=1}^I \epsilon^i l_t^i, \quad \forall t, \quad (4)$$

where  $c_t^i$  denotes consumption of an individual of age  $i$  at time  $t$ ,  $\epsilon^i$  denote her efficiency units, and  $l_t^i$  is hours worked.

The government in this economy finances an exogenous sequence of expenditure  $\{G\}_{t=0}^\infty$  using proportional capital taxes  $\theta_t$ , consumption taxes  $\eta_t$ , labor taxes  $\tau_t$  and debt  $D_t$ . The government intertemporal budget constraint is

$$G_t + R_t D_t \leq \eta_t C_t + \tau_t w_t L_t + \theta_t r_t K_t + D_{t+1}, \quad \forall t, \quad (5)$$

where  $R_t$  denotes the return on government debt. Let  $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^\infty$  be a tax policy consisting of an infinite sequence of proportional taxes and government debt, where  $D_0$  is given at  $t = 0$ . We rule out Ponzi schemes by imposing the transversality condition on government debt. In principle we are not imposing any bound on the feasible taxation.<sup>4</sup> The period resource constraint is

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq F(K_t, L_t), \quad \forall t. \quad (6)$$

Each generation has an endowment of one unit of time in each period and a life cycle profile of efficiency units of labor  $\epsilon = (\epsilon^1, \dots, \epsilon^I)$ . The endowment of an individual of age  $i$  can be transformed into  $\epsilon^i$  units of input in the production function. Households in this economy have standard preferences defined over a stream of consumption and labor/leisure and are represented by a time separable utility function

$$\prod_{i=1}^I \beta^{i-1} U(c_{t+i}^i, l_{t+i}^i), \quad \forall t, \quad (7)$$

where  $\beta > 0$  is the subjective discount factor,  $c_{t+i}^i$  and  $l_{t+i}^i$  represent the consumption and the time devoted to work by an individual of age  $i$  at time  $t + i$ . The utility function  $U : R_+^2 \rightarrow R$  satisfies  $U_c, -U_{cc} > 0$  and  $U_l, U_{ll} < 0$ . At each period, taking prices and taxes as given, individuals choose consumption, labor supply, and asset holdings. The consumer's at each period maximizes (7) subject to

$$(1 + \eta_t)c_t^i + a_{t+1}^{i+1} \leq (1 - \tau_t)w_t \epsilon^i l_t^i + (1 + r_t(1 - \theta_t))a_t^i, \quad 1 \leq i \leq I \quad \forall t, \quad (8)$$

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<sup>4</sup>This assumption is not innocuous and in particular the results stated in Proposition 2 depend on non-existence of upper bounds on capital income taxes.

$$a_t^1 = 0, \quad 0 \leq l_t^i \leq 1, \quad c_t^i \geq 0, \quad \forall t.$$

Each generation is born with no assets, and can accumulate wealth  $a_{t+1}^{i+1}$  by buying on-period government debt and lending to firms. Markets are complete, so different generations can intertemporally trade assets to smooth consumption over the life-cycle.<sup>5</sup>

At the initial period,  $t = 0$ , the stock of capital and debt is distributed among the initial,  $s$ , generations (individuals of age 2 to  $I$ ). Let  $\bar{a}_0^s = (k_0^s + d_0^s)$  be the endowment of capital and debt distributed among the initial generations, where  $K_0 = \sum_s k_0^s$  and  $D_0 = \sum_s d_0^s$ . The period-0 budget constraint is

$$(1 + \eta_0)c_0^s + a_1^{s+1} \leq (1 - \tau_0)w_0\epsilon^s l_0^s + (1 + r_0(1 - \theta_0))\bar{a}_0^s, \quad 2 \leq s \leq I. \quad (9)$$

Market clearing conditions in the capital markets imply

$$K_{t+1} = \sum_{i=1}^I a_{t+1}^i - D_{t+1}, \quad \forall t. \quad (10)$$

Next we proceed by defining the notion of competitive equilibrium.

**Definition 1 (Competitive Equilibrium):** *Given a tax policy  $\pi$  and a sequence of government expenditure  $\{G_t\}_{t=0}^\infty$ , a competitive equilibrium in this economy is a sequence of individual allocations  $\{\{c_t^i, l_t^i, a_{t+1}^i\}_{i=1}^I\}_{t=0}^\infty$ , production plans  $\{K_t, L_t\}_{t=0}^\infty$ , government debt  $\{D_{t+1}\}_{t=0}^\infty$ , and relative prices  $\{r_t, w_t, R_t\}_{t=0}^\infty$ , such that:*

1. *Generations born at time  $t \geq 0$  maximize (7) subject to (8). Similarly the initial generations born at  $t \leq 0$  maximize utility subject to (9).*
2. *In the production sector (1) and (2) are satisfied for all  $t$ .*
3. *Factor markets (4) and (10) clear.*
4. *The government budget constraint (5) is satisfied.*
5. *Feasibility (6) is satisfied for all  $t$ .*

### 3 Government problem

With the behavior of the market economy described in the previous section, we turn to the problem faced by the government. The objective of the

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<sup>5</sup>This assumption implies that agents are not credit constrained. Aiyagari (1995) shows that in an economy with uninsurable income risk the optimal capital tax is positive. The basic intuition works as follows. Because of the incomplete insurance markets, there is a precautionary motive for accumulating capital. In addition, the possibility of being borrowing constrained in the future leads to some additional savings. These facts increase the capital stock. Then, a positive capital tax is needed to reduce capital accumulation and equalize the interest rate of the economy to the rate of time preference.

government is to choose a tax policy  $\pi$  that maximizes the welfare of all (present and future) generations, and is consistent with the private sector equilibrium. As it has been pointed out by Kydland and Prescott (1977), the optimal policy might be time-inconsistent because the government can have incentives to deviate from the announced policy. In this paper we abstract from these issues and it is assumed that the government can commit to future policies.

In a representative consumer model, the social welfare function for the benevolent government has to be the consumer's utility function. In an economy with an infinite number of generations, the government needs to assign weight to the different consumers, and these weights are somewhat arbitrary. Let  $\omega_t \in R_+$  be the weight of a generation born at time  $t$ . In order to have a well-defined problem it is necessary to assume that the sequence of weights  $\{\omega_t\}_{t=-(I-1)}^\infty$  is summable,  $\sum_{t=-(I-1)}^\infty \omega_t < \infty$ . Formally the government objective function is

$$W(\{c_t^i, l_t^i\}) = \prod_{t=0}^\infty \prod_{i=1}^I \omega_{t+1-i} \beta^{i-1} U(c_t^i, l_t^i). \quad (11)$$

To find an asymptotic steady state for the government problem it is necessary to impose some structure on the sequence of weights<sup>6</sup>, such as  $\lim_{t \rightarrow \infty} \frac{\omega_t}{\omega_{t+1}} = \frac{1}{\lambda} > 1$  where  $\lambda \in (0, 1)$  is the relative weight between of present and future generations. Specifying the social welfare function to be of this form imposes some restrictions, because it rules out steady state "golden-rule" equilibria, as in Samuelson (1958).

The government problem of choosing the optimal policy is solved using the so-called *primal approach*, developed in Atkinson and Stiglitz (1980). The next proposition describes how to characterize the set of implementable allocations for a given tax policy  $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^\infty$ .

**Proposition 1 (Set of Implementable Allocations):** *Given a tax policy  $\pi$  a competitive equilibrium allocation  $x = \{\{c_t^i, l_t^i\}_{i=1}^I, K_{t+1}\}_{t=0}^\infty$  satisfies the following set of conditions:*

*i) period resource constraint:*

$$\prod_{i=1}^I c_t^i + K_{t+1} - (1 - \delta)K_t + G_t = F(K_t, \prod_{i=1}^I l_t^i), \quad \forall t, \quad (12)$$

*ii) implementability constraints for all newborn generations:*

$$\prod_{i=1}^I \beta^{i-1} c_{t+i-1}^i U_{c_{t+i-1}^i} + l_{t+i-1}^i U_{l_{t+i-1}^i} = 0, \quad t \geq 0, \quad (13)$$

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<sup>6</sup>Using a two period overlapping generations model, Atkinson and Sandmo (1980) derive the steady state optimal capital tax for different government discount factors. It can be easily shown that all are particular cases of this formulation.

iii) implementability constraints for the initial old generations at  $t = 0$ :

$$\prod_{i=s}^I \beta^{i-s} c_{i-s}^i U_{c_{i-s}^i} + l_{i-s}^i U_{l_{i-s}^i} = U_{c_0^s} a_0^s, \quad s = 2, \dots, I, \quad (14)$$

iv) marginal rates of substitution between consumption and labor, and consumption today and tomorrow are equal across consumers:

$$\frac{U_{c_t^1} \epsilon^1}{U_{l_t^1}} = \dots = \frac{U_{c_t^I} \epsilon^I}{U_{l_t^I}}, \quad i = 1, \dots, I, \quad (15)$$

$$\frac{U_{c_t^1}}{U_{c_{t+1}^2}} = \dots = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}}, \quad i = 1, \dots, I, \quad (16)$$

Furthermore, given allocations that satisfies (12), (13), (14), (15), and (16), we can construct a tax policy  $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^\infty$  and relative prices  $\{r_t, w_t, R_t\}_{t=0}^\infty$ , that together with the allocation  $x$ , constitute a competitive equilibrium.

**Proof.** See Appendix ■

In a representative consumer economy, the set of implementable allocations is uniquely determined by the period resource constraint and the implementability constraint. In an economy with heterogeneous consumers, these two conditions do not necessarily guarantee that the marginal rates of substitution across consumers are equal at a given period  $t$ . Unless specified, the government might find it optimal to tax different consumers with different tax rates. In this particular problem, the government might choose to condition taxes on age. Consider the following case where the government can use age-dependent taxes,  $\pi^i = \{\{\eta_t^i, \tau_t^i, \theta_t^i\}_{i=1}^I, D_t\}_{t=0}^\infty$ . The implementability constraints for each generation associated to the new tax system coincides with the one where the government cannot condition taxes on age, as in Erosa and Gervais (2002). Hence, if taxes cannot be conditioned on age, the set of implementable allocations has to include additional constraints to ensure that the marginal rates of substitution are equal across generations at a given period  $t$ .

Inspection of the first-order conditions of the consumer problem (displayed in the appendix) show that if an allocation belongs to the set of implementable allocations, then it can be decentralized under a variety of tax schemes.

**Corollary 1:** *Given a sequence of  $\{G\}_{t=0}^\infty$  and an initial distribution of wealth  $\{\bar{a}_0^s\}_{i=2}^I$ , if  $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^\infty$  is the tax policy associated to an implementable allocation  $x$ , then there exists another tax policy  $\pi' = \{\eta'_t, \tau'_t, \theta'_t, D'_t\}_{t=0}^\infty$  that supports the same allocation.*

**Proof.** See Appendix ■

The primal approach implements optimal wedges between the marginal rates of substitution and marginal rates of transformation, but it does not prescribe any particular type of instruments. As a result, the optimal policy can be supported as a competitive equilibrium under a variety of tax schemes. Such a system could include those with only consumption and labor income taxes, or more complicated tax systems. In this particular paper we are interested in capital income taxation, so we set consumption taxes,  $\eta_t = 0$ , in all periods.

In an infinitely-lived consumer economy, the government has incentives to tax heavily the initial stock of capital at  $t = 0$  and achieve a Pareto efficient allocation. To avoid an effective lump-sum tax, it is generally assumed that the government takes as given the initial capital tax  $\theta_0$  or that there exists an upper bounds on the feasible capital income tax. In an overlapping generations economy, the individuals that face a front-loading tax policy are different than the ones that benefit from the reduction of distortionary taxes in the future. As a result the government faces a trade off between efficiency and intergenerational redistribution. Hence, a tax in the initial distribution of wealth  $\{\bar{a}_0^s\}_{s=0}^I$  is equivalent to lump-sum tax only on the initial generations alive at  $t = 0$ . Through out the paper we assume that the initial capital tax  $\bar{\theta}_0$  is given.

The government problem is to maximize the social welfare function over the set of implementable allocations. Formally,

$$\max_{\{c_t^i, l_t^i\}_{i=1}^I, K_{t+1}} \prod_{t=0}^{\infty} \prod_{i=1}^I \omega_{t+1-i} \beta^{i-1} U(c_t^i, l_t^i)^\alpha, \quad (17)$$

$$s.t. \quad \prod_{i=1}^I c_t^i + K_{t+1} - (1 - \delta)K_t + \bar{G}_t = F(K_t, \prod_{i=1}^I \epsilon^i l_t^i), \quad \forall t, \quad (18)$$

$$\prod_{i=1}^I \beta^{i-1} c_{t+i-1}^i U_{c_{t+i-1}^i} + l_{t+i-1}^i U_{l_{t+i-1}^i} = 0, \quad t \geq 0, \quad (19)$$

$$\prod_{i=s}^I \beta^{i-s} c_{i-s}^i U_{c_{i-s}^i} + l_{i-s}^i U_{l_{i-s}^i} = U_{c_0^s} a_0^s, \quad s = 2, \dots, I, \quad (20)$$

$$\frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}} = \dots = \frac{U_{c_t^{I-1}}}{U_{c_{t+1}^I}}, \quad i = 1, \dots, I, \quad (21)$$

$$\frac{U_{c_t^1}}{U_{l_t^1}} \epsilon^1 = \dots = \frac{U_{c_t^I}}{U_{l_t^I}} \epsilon^I, \quad i = 1, \dots, I. \quad (22)$$

where the initial distribution of wealth  $a_{s,0}$ ,  $K_0 = \bar{K} > 0$  and  $\{G_t\}_{t=0}^{\infty}$  are given and  $c_t^i \geq 0$ ,  $l_t^i \in [0, 1]$ .

The allocation  $x$  that solves the government problem is constrained efficient, in the sense that there exists no other constrained efficient allocation

$x'$  belonging to the set of implementable allocations that dominates the optimal.

To solve the government problem we consider a relaxed version of the problem with the constraints (21) and (22) dropped. If the solution of the relaxed version satisfies the constraints dropped, then it solves the original problem. Then, we look for a sufficient condition on preferences that implies zero capital taxes and also satisfies the additional constraints (21) and (22). To derive the first-order conditions, it is useful to redefine the Lagrangian by introducing the implementability constraint on it. Let  $\mu_{t-i}$  be the Lagrange multiplier of the implementability constraint<sup>7</sup> for the agent born in period  $t - i$ . Then, let's define

$$W(c_t^i, l_t^i, \mu_{t-i}) = U(c_t^i, l_t^i) + \mu_{t-i}(c_t^i U_{c_t^i} + l_t^i U_{l_t^i}). \quad (23)$$

the additional term measures the effect of distortionary taxes on the utility function. In particular, it captures the effect of the distortion on the marginal rate of substitution. If the implementability constraint binds, the first-order conditions of the consumer problem are distorted unless the term in the parenthesis is equal to zero. With the new notation the government problem is given by

$$\begin{aligned} \max_{\{c_t^i, l_t^i\}_{i=1}^I, K_{t+1}} \prod_{t=0}^{\infty} \prod_{i=1}^I \omega_{t+1-i} \beta^{i-1} W(c_t^i, l_t^i, \mu_{t-i}) - \prod_{s=2}^I \mu_{1-s} U_{c_0^s} a_0^s, \\ \text{s.t.} \quad \prod_{i=1}^I c_t^i + K_{t+1} - (1 - \delta)K_t + G_t = F(K_t, \prod_{i=1}^I \epsilon^i l_t^i), \quad \forall t. \end{aligned} \quad (24)$$

Let  $\psi_t$  be the Lagrange multiplier of the resource constraint, then the first-order necessary conditions for an interior solution at  $t > 0$  are

$$\begin{aligned} [c_t^i] \quad \omega_{t+1-i} \beta^{i-1} W_{c_t^i} - \psi_t &= 0, \\ [c_t^{i+1}] \quad \omega_{t-i} \beta^i W_{c_t^{i+1}} - \psi_t &= 0, \\ [l_t^i] \quad \omega_{t+1-i} \beta^{i-1} W_{l_t^i} + \psi_t F_{L_t} \epsilon^i &= 0, \\ [K_{t+1}] \quad -\psi_t + \psi_{t+1}(1 - \delta + F_{K_{t+1}}) &= 0, \end{aligned}$$

together with the period resource constraint (25) and the transversality condition for the optimal capital path

$$\lim_{t \rightarrow \infty} \psi_t K_{t+1} = 0. \quad (26)$$

Throughout the paper we assume that the solution of the Ramsey allocation problem exists and that the time paths of the solutions converge to a steady

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<sup>7</sup>If the government has access to lump-sum taxes, the implementability constraint will not be binding,  $\eta_{t-i} = 0$ , and it will not be optimal to use distortionary taxes and the economy would achieve a full efficient allocation.

state. Neither of these assumptions is innocuous. The sufficient conditions for an optimum involve third derivatives of the utility function. Therefore, the solutions might not represent a maximum, or the system might not have a solution because it does not exist a feasible policy that satisfies the intertemporal government budget constraint. However, assuming that the solution to the government problem exists and is interior, it will satisfy the above first-order conditions. Hence, the optimal taxation analysis will apply to these cases only. Rearranging terms, we have

$$\omega_{t+1-i}W_{c_t^i} = \omega_{t+2-i}W_{c_{t+1}^i}(1 - \delta + F_{Kt+1}), \quad \forall t, i, \quad (27)$$

$$\frac{W_{l_t^i}}{W_{c_t^i}} = -F_{L_t}\epsilon^i, \quad \forall t, i, \quad (28)$$

$$W_{c_t^i} = \frac{\omega_{t-i}}{\omega_{t+1-i}}\beta W_{c_{t+1}^i}, \quad \forall t, i. \quad (29)$$

Notice that Equation (27) is slightly different from the consumer Euler equation. The government equates the derivative of the objective function of a newborn generation at different times. Equation (28) is the intratemporal condition between consumption and labor, that determines the amount of effective hours worked by each generation at a given period  $t$ . Finally, equation (29) is the static redistributive condition, and implies that the government will assign consumption among two different generations according to the ratio of their relative weights. This condition does not appear in the equilibrium conditions for the private sector, but it is very useful to derive the optimal capital taxes. Updating (29) one period and substituting in (27) we obtain

$$W_{c_t^i} = \beta W_{c_{t+1}^i}(1 - \delta + F_{Kt+1}), \quad \forall i, t, \quad (30)$$

This new expression resembles a life-cycle Euler equation, but instead of having the marginal utility with respect to consumption involves the derivative of  $W(\cdot)$  with respect to consumption. For the initial  $s$  generations at  $t = 0$ , the first-order necessary conditions are incorporate the initial distribution of asset holdings. Formally,

$$\frac{W_{l_0^s} - \mu_{1-s} U_{c_0^s l_0^s} (1 + F_{K_0}(1 - \bar{\theta}_0)a_0^s + U_{c_0^s} F_{K_0^s} (1 - \bar{\theta}_0)a_0^s)}{W_{c_0^s} - \mu_{1-s} U_{c_0^s c_0^s} (1 + F_{K_0}(1 - \bar{\theta}_0)a_0^s)} = -F_{L_0}\epsilon^s, \quad (31)$$

$$\frac{W_{c_0^s} - \mu_{1-s} U_{c_0^s c_0^s} (1 + F_{K_0}(1 - \bar{\theta}_0)a_0^s)}{W_{c_0^{s+1}} - \mu_{2-s} U_{c_0^{s+1} c_0^{s+1}} (1 + F_{K_0}(1 - \bar{\theta}_0)a_0^{s+1})} = \beta \frac{\omega_{s+1}}{\omega_s}. \quad (32)$$

To derive the optimal tax policy  $\pi^*$  we have to substitute the allocations obtained from the government problem into the private sector equilibrium

conditions. The resulting expressions are the optimal capital and labor tax rate are

$$\theta_{t+1}^* = \frac{1}{\beta r_{t+1}} \frac{W_{c_t^i}}{W_{c_{t+1}^{i+1}}} - \frac{U_{c_t^i}}{U_{c_{t+1}^{i+1}}}, \quad \forall t. \quad (33)$$

$$\tau_t^* = 1 - \frac{U_{c_t^i} W_{l_t^i}}{U_{l_t^i} W_{c_t^i}}, \quad \forall t. \quad (34)$$

If we drop the age subscripts from the first-order conditions of the government problem, the associated expressions for the optimal tax policy are equivalent to the ones obtained in an infinite-lived consumer economy. Judd (1985, 1999) and Chamley (1986) prove two results for this class of economies. First, for a general class of utility functions capital taxes should be zero in the long run (consumption is constant, therefore  $U_{c_t} = U_{c_{t+1}}$ ). Second, for a particular class of functions, that satisfy  $W_{c_t}/W_{c_{t+1}} = U_{c_t}/U_{c_{t+1}}$ , the optimal capital income taxes are zero after a finite number of periods. The conditions for the zero capital result in the transition path are generally viewed as an application of the uniform commodity taxation principle (see Atkinson and Stiglitz (1980)), that specifies conditions under which taxing all goods at the same rate is optimal.

In overlapping generations economies when the government cannot use age-dependent taxes the previous results are not generally true. Notice that the marginal utility is not constant over the life-cycle, even in steady state.

As a result, we cannot expect zero taxes on capital returns unless two conditions are satisfied. First, for a given utility function the ratio  $W_{c_t^i}/W_{c_{t+1}^{i+1}}$  must to be equal to the solution of the private sector equilibrium  $U_{c_t^i}/U_{c_{t+1}^{i+1}}$ . Second, given that we are solving a relaxed version of the government problem, we have to ensure that constraints (21) and (22) are satisfied as well.<sup>8</sup> The next proposition provides a sufficient condition on consumer utility function that guarantees the zero capital income tax result. Unfortunately, most preferences do not satisfy the condition.

**Proposition 2:** *When the government cannot condition taxes on age, that is taxes are age-independent, if preferences satisfy*

$$\frac{c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i}}{U_{c_t^i}} = \frac{l_t^i U_{ll_t^i} + c_t^i U_{cl_t^i}}{U_{l_t^i}}, \quad \forall t > 1 \quad (35)$$

*then the optimal capital and labor income tax are zero for  $t \geq 2$ .*

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<sup>8</sup>Clearly, if the first condition is satisfied, the set of constraint (21) is also satisfied but it does not imply that constraints (22) are satisfied too. Hence, if the government cannot condition labor income taxes on age, the restrictions on the set of instruments have an important impact on the sufficient conditions the zero capital income tax result.

**Proof.** We need to show two results under this condition. First, that the optimal capital income tax is zero from  $t \geq 2$ . Second, that if preferences satisfy this property, then the solution of the less constrained government problem is also a solution to the more constrained problem. We proceed by rewriting condition (35) as follows

$$c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i} = AU_{c_t^i}, \quad (36)$$

$$l_t^i U_{ll_t^i} + c_t^i U_{cl_t^i} = AU_{l_t^i}. \quad (37)$$

where  $A$  is a constant. Now let's consider the first-order conditions of the government problem with respect to  $c_t^i$

$$(1 + \mu_{t-i})U_{c_t^i} + \mu_{t-i} \left[ c_t^i U_{cc_t^i} + l_t^i U_{lc_t^i} \right] = \psi_t,$$

where  $\mu_{t-i}$  and  $\psi_t$  denote the Lagrange multipliers of the implementability constraint of a generation born at period  $t-i$  and the period  $t$  resource constraint respectively. Substituting Equation (36) in the first order conditions, we can rewrite the expression as

$$U_{c_t^i}(1 + \mu_{t-i}(1 + A)) = \psi_t, \quad (38)$$

since this equations holds for time  $t$  and  $t+1$ , then we combine the equation with the first-order condition with respect to capital and obtain

$$\frac{U_{c_t^i}}{\beta U_{c_{t+1}^{i+1}}} = 1 + F_{Kt+1} - \delta.$$

Clearly all consumer face the same prices, hence constraint (21) is satisfied if the utility function satisfies (36).<sup>9</sup> Now we want to show that condition (36) together with condition (37) are sufficient to ensure that the solution of the less constrained problem is a solution of the more constrained problem. Using the same argument we have

$$U_{l_t^i}(1 + \mu_{t-i}(1 + A)) = -\psi_t F_{L_t} \epsilon^i. \quad (39)$$

Since Equations (38) and (39) hold for all generations at a given period  $t$ , then the marginal rates of substitution between consumption and labor are equal across generations. So, constraint (22) is also satisfied.

At  $t = 1$ , capital and labor taxes are different from zero because the implementability constraints of the initial generations include the initial distribution of capital stock, that prevents capital taxes from being zero. At

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<sup>9</sup>This condition is sufficient to ensure zero capital taxes in an infinitely-lived consumers model, but in these type of economies it does not guarantee that the additional constraints on the marginal rates of substitution between consumption and leisure are satisfied.

the initial period  $t = 0$  the capital taxes are given,  $\bar{\theta}_0$ . Under this policy the government only collects taxes at  $t = 0, 1$ , but it is able to affect the cumulative discount rate, with the initial taxes on capital. ■

A key assumption for the result is the non-existence of an upper bound on feasible capital income taxes,  $\theta \leq 1$ . As a result, the optimal capital income tax at  $t = 1$  during those period is effectively taxing the wealth of the initial generations. Imposing bounds on the optimal tax rate modifies this result because the government problem has to incorporate additional constraints<sup>10</sup> that would imply some initial periods with positive taxation.

Nevertheless, the importance of this result is to show that the standard preferences defined over consumption and leisure<sup>11</sup> commonly used to evaluate fiscal policies as

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} + v(1-l), \quad (40)$$

or

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} v(1-l), \quad (41)$$

do not satisfy the sufficient condition. For these two cases the optimal policy might imply capital income taxes different from zero. However, Escolano's quantitative results show that preferences that do not satisfy the sufficient conditions can be consistent with zero capital income taxes for some plausible parameter values.

Inspection of the sufficient condition gives some insight of requirements that preferences need to satisfy to ensure the zero capital income result. For a class of additively separable, the sufficient condition requires a constant elasticity of the marginal utility and that it is not satisfied by preferences that are non-homothetic with respect to labor. Consider utility functions  $U(c, l)$  where  $c$  and  $l$  are homothetic. We then have

**Proposition 3:** *If the utility function is homothetic with respect to consumption and hours worked, then the steady state capital tax is zero.*

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<sup>10</sup>If the feasible capital tax has to satisfy  $\tau_t^k \leq 1$ , we have to impose an additional constraint to the government problem

$$U_{c_t^i} - \beta U_{c_{t+1}^{i+1}}(1-\delta) \geq 0, \quad \forall t,$$

In general, it can be shown that this constraint only binds a finite number of periods  $T^*$ . As a result the optimal capital and labor income taxes in this economy will only be zero from  $T^* + 1$  onwards.

<sup>11</sup>It is important to remark that  $l$  denotes hours worked and  $(1-l)$  is leisure. If we redefine the utility function to depend on leisure  $l$  and  $(1-l)$  hours worked, then the associated implementability constraint has to be modified to include  $(1-l)U_l$  instead of  $lU_l$ . This modification does not change the previous results.

**Proof.** This class of preferences satisfies

$$cU_{cc} + lU_{lc} = AU_c, \quad (42)$$

$$lU_{ll} + c^iU_{cl} = BU_l. \quad (43)$$

where  $A$  and  $B$  are different constants. Next, we prove that under these assumptions the optimal labor taxes satisfies the additional constraint (21) in steady state. In this case, the first-order conditions of the government problem are

$$\frac{(1 + \mu)U_{c^i} + \mu[c^iU_{cc^i} + l^iU_{lc^i}]}{(1 + \mu)U_{l^i} + \mu[l^iU_{ll^i} + c^iU_{cl^i}]} = \frac{1}{F_L e^i}, \quad (44)$$

where  $\mu$  the Lagrange multiplier of the implementability constraint is constant.<sup>12</sup> All newborn generations face the same prices and taxes over the life-cycle. Substituting Equation (42) and (43) into the first-order conditions, we can rewrite Equation (44) as

$$\frac{U_{c^i}}{U_{l^i}} = \frac{(1 + \mu(1 + B))}{(1 + \mu(1 + A))} \frac{1}{w e^i}, \quad (45)$$

where  $1 - \tau = (1 + \mu(1 + A))/(1 + \mu(1 + B))$  is the optimal labor tax. Since this expression holds across generations, the additional constraint (22) are satisfied. Outside the steady state, the Lagrange multiplier of the implementability constraint are not constant across generations because the relative prices change along the transition path. ■

An example of utility function that satisfies this property is

$$U(c, l) = \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\phi}}{1+\phi} \quad (46)$$

where  $A = -\sigma$  and  $B = \phi$  are both constants. For a positive Lagrange multiplier of the implementability constraint,  $\mu$ , the optimal labor tax is positive.

In general, the conditions under which capital income taxes are zero are viewed as an application of the uniform commodity taxation principle. In overlapping generation economies, if labor is non-homothetic and the government cannot use age-dependent taxes, we cannot expect capital taxes to be zero. From the government perspective labor supply of different generations is viewed as a different commodity, so it has an incentive to tax it when it is relatively more inelastic. When taxes cannot be conditioned on age, the government uses a non-zero capital tax as an indirect instrument that can be used to tax leisure. In general, there is no reason to expect

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<sup>12</sup>The Lagrange multipliers have been previously normalized by the government discount rate.

optimal uniform labor income taxes. Even when leisure is homothetic, it is optimal to tax it indirectly through non-uniform taxation. In an overlapping generations model, the distortions associated with this policy are not that important because for a given generation today's consumption and period  $T$  consumption are not perfectly substitutable. Hence the effect of capital distortions is much smaller and not necessarily bigger than distortions caused by other taxes like labor income taxes.

We believe that these results improve the existing literature in several ways. First, it considers a general model where individuals live  $I$  periods and taxes cannot be conditioned on age. Second, the theoretical analysis includes the transition path as well as the steady state. So, these results provide a theoretical foundation of the quantitative findings in Escolano's (1992), and are complementary to the findings of Erosa and Gervais (2002). In general we find that in this class of models considering only two period lived-agents can generate misleading results. When the government cannot condition taxes on age, the additional constraints that this restriction imposes in the set of tax instruments plays an important role in the determination of the optimal policy. We summarize this finding in the next proposition,

**Proposition 4:** *The uniform commodity tax condition (36) is a sufficient condition to ensure zero capital taxes in steady state and also along the transition path from period  $t \geq 2$  if*

- 1) *The government has access to age-dependent taxes, or*
- 2) *generations live two periods and the old does not supply labor.*

**Proof.** If the government can use age-specific taxes, then the government problem ignores the additional constraints (21) and (22). Therefore, the optimal policy for utility functions of the form (40) and (41) imply zero capital taxes from period two onwards. For these cases the age-dependent labor taxes are different from zero. Equivalently, in a two period OLG economy where the old generation does not supply labor, the government problem does not include additional constraints of the form (21) and (22). Usually, the young generation supplies labor<sup>13</sup> in the market while the old generation supplies capital. ■

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<sup>13</sup>One might think that one way to get around the non-zero capital tax result is to consider inelastic labor supply. With inelastic labor supply the government can ignore the constraints (22). Next, we show in a simple two period model that this intuition is not correct. The implementability constraint associated with the problem would be  $c_{1t}U_{c1t} + \beta c_{2t+1}U_{c2t+1} = U_{c1t}w_t$ . From the firm problem we know that  $w_t = f(k_t) - k_t f'(k_t)$ . Given that the government problem can only depend on allocations, the IC should be written as  $c_{1t}U_{c1t} + \beta c_{2t+1}U_{c2t+1} = U_{c1t}(f(k_t) - k_t f'(k_t))$ . Then, even if preferences satisfy the uniform commodity taxation condition the implied capital income taxes are different from zero, unless the effective lump-sum labor tax is sufficient to finance government expenditure. Obviously, in this case the optimal capital income tax is zero.

Next, to confirm the connection between commodity taxation and capital income taxation we simulate the model for an empirically reasonable intertemporal utility function and a robust demographic structure.

## 4 Quantitative Results

### 4.1 Parameterization

This section describes the choice of the functional forms for the numerical simulations and the parameterization process. The functional forms are chosen to have comparable results with Chari and Kehoe (1999). The utility function is

$$U(c, l) = \frac{(c^\gamma(1-l)^{1-\gamma})^{1-\sigma} - 1}{1-\sigma},$$

where  $\gamma \in (0, 1)$  is the consumption share in the utility function and  $\sigma$  denotes the inverse of the intertemporal elasticity of substitution. In the benchmark economy we set  $\sigma = 1$  as Chari and Kehoe (1999). In his quantitative analysis, Escolano (1992) uses a CES utility function. Clearly, neither type of utility function satisfies the sufficient conditions for zero capital income taxes. The aggregate technology is a Cobb-Douglas production function,  $F(K, N) = K^\alpha N^{1-\alpha}$ .

A period in the model is one year, and we assume that agents live 59 periods. Hence, the model can be interpreted as one in which individuals are born economically at age 20 and live up to a maximum of 79 years. The empirical evidence shows that hours worked are not constant over the life-cycle. Ghez and Becker (1975) and Juster and Stafford (1991) find that households allocate one third of their discretionary time in market activities. Setting  $\gamma = 0.4$  in the model implies that individuals work an average of 33% of their time endowment over the life-cycle. The discount factor  $\beta$  is chosen together with the intertemporal elasticity of substitution to match the observed capital/output ratio of the economy. Given that in the benchmark economy  $\sigma = 1$ , setting  $\beta = 0.99$  matches the observed average capital/output ratio of 2.4 for the market economy. The labor earnings age profile  $\{\epsilon_j\}$  is derived using PSID data. In the model, labor services are homogeneous, so there is a single wage per efficiency unit of labor. Hence,  $\{\epsilon_j\}$  is chosen to match the age profile of average wages in the cross-section of US data.

In the technology, the estimate of  $\alpha = 0.33$  comes from the computation of capital's share in national income, which includes durables as a part of the capital stock. The aggregate depreciation rate,  $\delta$ , depends on the aggregate investment/output ratio. Given that we do not explicitly include growth this number must be larger to match investment. A value of  $\delta = 0.08$  matches

the observed average investment-output ratio of 16.1%. The capital/output ratio together with the depreciation rate imply a gross interest rate of 5.6%. The government consumption is set to 19%, which corresponds to the average of the last decade. In determining taxation on factor earnings, we follow the methodology of Mendoza, Tesar and Razin (1994). They develop consistent measures of the effective tax rate on factors' income for OECD countries. The competitive economy is calibrated using the last decade average tax rates, which imply a capital income tax of 35% and a labor income tax of 24%. Given  $G/Y$  and the tax policy  $\pi$ , the ratio  $D/Y$  is endogenously determined by the model. The resulting debt/output ratio in the market economy is 24%. This figure is roughly consistent with the average observed in the last few decades, which is 23%.

## 4.2 Findings

In this section we start by computing the initial steady state equilibrium for the market economy. Then, we assume that the government implements the optimal policy and the economy converges to a new steady state.<sup>14</sup> We then quantify the optimal capital and labor income taxes for two different tax regimes (age-independent and age-dependent taxes) and then we establish the connection between commodity taxation and capital income taxation. To compute the optimal policy we need to specify a value for the relative weight that the government places between present and future generations. This parameter has no counterpart in the market economy and the quantitative results can drastically change for different values. We then choose to compare the results for a broad range of values for this parameter.

Table 1 presents a summary of the steady state results obtained for the case where the government cannot condition taxes on age. The market equilibrium for the benchmark economy is displayed in the last row (ME).

[Insert Table 1]

As one should expected the quantitative findings are consistent with Escolano (1992). We briefly summarize the main quantitative features of the solution. The parameterized model can justify the observed tax rates for capital and labor income for some plausible values of the government discount rate. In particular for values of  $\lambda$  that are consistent with the observed capital/output ratio, the implied optimal policy predicted by the model is very similar to the benchmark taxation displayed in the last row. Changes in  $\lambda$  have important redistributive effects because the government changes

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<sup>14</sup>A feature of this model is that if the economy converges to the steady state, then it has the modified golden rule property and it is independent of the initial conditions (see Escolano (1992)).

its relative weight between the young and old generations. If the government increases  $\lambda$  it lowers the gross interest rate of the economy in steady state (given by  $\frac{1}{\lambda} = 1 - \delta + \alpha Y/K$ ) and increases the capital/output ratio. The optimal tax on capital returns is inversely related with the government discount factor. For some parameter value,  $\lambda = 0.97$ , the optimal capital income tax is roughly zero, and for higher values the government subsidizes investment. The optimal level of debt is also inversely related with  $\lambda$ . For some values of  $\lambda$  the government owns part of the capital stock and lends to firms. Another parameter that can have important effects on the optimal policy is the intertemporal elasticity of substitution. Table 2 summarizes the results for different values of  $\sigma$ .

[Insert Table 2]

We observe that changes in the intertemporal elasticity of substitution have important effects on the optimal capital income tax. As we increase  $\sigma$ , households have a higher preference for consumption smoothing, and to reduce the distortions the government responds by lowering the optimal capital income tax.

Next, we analyze the optimal policy when the government can condition taxes on age. Formally, that implies dropping constraints (21) and (22) from the government problem. Figure I presents a summary of the results for the optimal policy in the benchmark economy where we set  $\lambda = 0.947$  and  $\sigma = 1$ .

[Insert Figure I]

For the benchmark case, the age-specific capital tax is constant across households and equal to zero, as predicted by Erosa and Gervais (2002). This result is consistent with Proposition 3 because the utility function satisfies the uniform commodity taxation principle.<sup>15</sup> Since consumption and leisure move together over the life-cycle, the government has an incentive to tax labor when is relatively more inelastic. That occurs in the periods of the life-cycle where households are working relatively more. The hump in the distribution of labor taxes occurs at the ages where the efficiency units of labor also exhibit a hump. If the profile of efficiency units is flat, the optimal labor tax policy would imply a decreasing labor income tax over the life-cycle. Changes in the government discount rate do not affect the optimal tax on capital returns. Then, it is important to remark that the zero

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<sup>15</sup>In this case the additional conditions that restrict the marginal rates of substitution between consumption today and tomorrow are not binding. In order to satisfy the sufficient conditions for zero capital taxation we must allow labor taxes to differ across agents.

capital income tax result is obtained independently of the relative weight that the government places on present and future generations. Changes in  $\lambda$  only affect the distribution of labor income taxes mainly due to the effect on the relative prices. The level of debt is adjusted to satisfy the desired capital/output ratio. For a higher values of the intertemporal elasticity of substitution the optimal capital tax across generations is different from zero. Figure 2 displays the optimal age-specific taxes for different values of  $\sigma$ .

[Insert Figure II]

With non-separable preferences the optimal capital tax across ages implies subsidizing the asset returns of the younger generations and taxing at a positive rate the asset return of the older ones. For this particular functional forms, an increase in  $\sigma$  does not substantially affect the distribution of capital taxes across generations, but it lowers the labor-specific taxes for all ages.

Finally, we explore the connection between commodity taxation and capital income taxation. For a given age-specific capital income tax there exists an equivalent consumption tax can be computed using  $(1 + \eta_i)/(1 + \eta_{i+1}) = (1 + r(1 - \theta_i))/(1 + r)$  for all  $i$ . If the age-specific capital tax is zero, then the optimal consumption tax is also zero. When capital taxes are positive (negative) for a specific age,  $\theta_i > 0$  ( $\theta_i < 0$ ) consumption taxes have to increase (decrease) over the life-cycle. Figure III compares the optimal consumption tax over the life-cycle. In order to have comparable results with the flat consumption tax, we have normalized  $\eta_0$  the initial tax to zero.

[Insert Figure III]

For the benchmark case, where the optimal capital income tax is  $\theta_i = 0$  for all  $i$ , the optimal consumption tax is  $\eta_i = 0$  for all  $i$ . For different values of the intertemporal elasticity of substitution, the life-cycle consumption tax is J-shaped. The capital income subsidy is equivalent to a decreasing consumption tax in the early stages of the life-cycle, that is then reverted when the capital income tax is positive. In the next stages of the life-cycle the consumption tax is increasing. This pattern confirms the basic intuition that consumption is taxed when is relatively high. When taxes cannot be conditioned on age, this non-constant consumption tax can be imperfectly replicated by setting a non-zero capital tax that modifies the relative Arrow-Debreu price for consumption goods.

## 5 Conclusions

This paper explores the proposition that the optimal capital income tax is zero. In contrast with previous studies, we consider an overlapping genera-

tions version of the neoclassical growth model to analyze the optimal fiscal policy along the transition path and in the long-run. In this context, we provide sufficient conditions for zero capital income tax result and we show that it is very difficult to obtain zero optimal capital taxes if the government cannot condition taxes on age. When the government cannot condition taxes on age, the additional constraints that this restriction imposes in the set of tax instruments plays an important role in the determination of the optimal policy. However, the uniform commodity taxation principle is a sufficient condition to ensure zero optimal capital taxes if either the government can condition taxes on age, or generations live two periods and the old does not supply labor.

To confirm the connection between commodity taxation and capital income taxation we simulate the model for an empirically reasonable intertemporal utility function and a robust demographic structure. The quantitative findings are consistent with Escolano (1992). If the government cannot condition taxes on age, given some plausible choices of parameter values, the optimal policy can be consistent with the observed tax rates. Nevertheless, the optimal capital tax predicted by the model can change when the government can condition taxes on age. For preferences consistent with the uniform commodity tax result the optimal capital tax across ages is zero. As in the infinite-lived consumer model, the zero capital income tax is equivalent to a zero consumption tax across ages too. With preferences that are not consistent with the uniform commodity tax result, we find that the government has an incentive to tax consumption when it is relatively high. This policy can be mimicked with a J-shaped consumption tax over the life-cycle. When taxes cannot be conditioned on age, this policy can be imperfectly replicated by setting a non-zero capital income tax.

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## 7 Appendix

**Proof of Proposition 1:** We first proceed by showing that the allocations in a competitive equilibrium must satisfy (12), (13), (14), (15), and (16). Condition (12) is straightforward from substituting the labor market clearing condition (4) into (6).

The implementability constraint for each generation is constructed by substituting the consumer first-order conditions

$$\frac{U_{l_t^i}}{U_{c_t^i}} = \frac{(1 - \tau_t)w_t \epsilon^i}{(1 + \eta_t)} \quad \forall t, i, \quad (47)$$

$$\frac{U_{c_t^i}}{(1 + \eta_t)} = \frac{\beta U_{c_{t+1}^{i+1}}}{(1 + \eta_{t+1})} (1 + r_{t+1}(1 - \theta_{t+1})) \quad \forall t, i. \quad (48)$$

into the intertemporal budget constraint

$$\times_{i=0}^{I-1} p_{t+i} (1 + \eta_{t+i}) c_{t+i}^i \leq \times_{i=0}^{I-1} p_{t+i} (1 - \tau_{t+i}) w_{t+i} \epsilon^i l_{t+i}^i, \quad (49)$$

where  $p_{t+i}$  denotes the Arrow-Debreu price for the consumption good at period  $t+i$ . We then use the definition of  $p_{t+i}$  to substitute for the interest rate in the intertemporal condition. For the initial generations in the economy at time  $t=0$  the distribution of asset holdings appears on the right hand side of the the budget constraint. That explains the additional term (14) on the implementability constraint. If the government is restricted to use the same proportional taxes for all generations the set of implementable allocations needs to include constraints (15), and (16).

Now we prove the second part of Proposition 1. Now we prove that given an allocation  $x$  that satisfies the previous conditions, it is possible to construct a sequence of prices and a policy  $\pi$  that together with the allocation constitute a competitive equilibrium. From the aggregate capital stock,  $K_t$ , and the aggregate labor supply,  $L_t$ , we construct the relative prices using the firm's first-order conditions (1) and (2). To derive the government policy  $\pi = \{\eta_t, \tau_t, \theta_t, D_t\}_{t=0}^{\infty}$ , we substitute the allocations  $\{\{c_t^i, l_t^i\}_{i=1}^I\}_{t=0}^{\infty}$  and the equilibrium prices  $\{r_t, w_t, \}_{t=0}^{\infty}$  into households' first-order conditions (47) and (48). To obtain, the intertemporal budget constraint for the households we have to substitute  $U_{c_t^i}$  and  $U_{l_t^i}$  into (13), (14). All these conditions determine a system of equations from where we obtain  $\pi$ . The sequence of government debt  $\{D_t\}_{t=0}^{\infty}$  is adjusted to satisfy the market equilibrium capital/output ratio

$$D_{t+1} = \times_{i=1}^I a_{t+1}^i - K_{t+1}. \quad (50)$$

Finally, combing feasibility and the households budget constraint for a given period the government budget constraint also has to be satisfied. For a given period  $t$ , add up all the generations budget constraint:

$$\times_{i=1}^I (1 + \eta_t) c_t^i - \times_{i=1}^I (1 - \tau_{t+i}) w_t \epsilon^i l_t^i \leq \times_{i=1}^I (1 + r_t(1 - \theta_t)) a_t^i - \times_{i=1}^I a_{t+1}^{i+1},$$

aggregating variables and using the market clearing condition in the capital market we obtain

$$\eta_t C_t + \tau_t w_t L_t + \theta_t r_t K_t + R_t B_t + B_{t+1} \leq F_{L_t} L_t + F_{K_t} K_t - C_t - K_{t+1} + (1 - \delta) K_t,$$

Combining the previous expression with the resource constraint we obtain the period government budget constraint. ■

**Proof of Corollary 1:** We want to show how different tax policies can be consistent with the same allocation  $x$ . Clearly, the relative prices  $\{r_t, w_t\}$  and the resource constraint depend uniquely on the allocation  $x$ . The two policies  $\pi$  and  $\pi'$  satisfy

$$\frac{(1 + \eta_t)}{(1 - \tau_t)} = \frac{(1 + \eta'_t)}{(1 - \tau'_t)} \quad \forall t, i, \quad (51)$$

and,

$$\frac{(1 + r_{t+1}(1 - \theta_t))(1 + \eta_t)}{(1 + \eta_{t+1})} = \frac{(1 + r_{t+1}(1 - \theta'_t))(1 + \eta'_t)}{(1 + \eta'_{t+1})} \quad \forall t, i, \quad (52)$$

because the relative prices haven't change. Substituting (51) and (52) in the households' budget constraint:

$$(1 + \eta_t)c_t^i - \frac{(1 - \tau'_t)(1 + \eta_t)}{(1 + \eta'_t)} w_t \epsilon^i l_t^i = (1 + r_t(1 - \theta'_t)) \frac{(1 + \eta'_{t-1})(1 + \eta_t)}{(1 + \eta'_t)(1 + \eta_{t-1})} a_t^i - a_{t+1}^{i+1}, \quad (53)$$

and multiplying in both sides by  $(1 + \eta'_t)/(1 + \eta_t)$  :

$$(1 + \eta_t)c_t^i - (1 - \tau'_t)w_t \epsilon^i l_t^i = (1 + r_t(1 - \theta'_t))\mathbf{e}_t^i - \mathbf{e}_{t+1}^{i+1} \quad \forall t, i,$$

where  $\mathbf{e}_t^i$  and  $\mathbf{e}_{t+1}^{i+1}$  are the equivalent distribution of asset holdings:

$$\mathbf{e}_t^i = \frac{(1 + \eta'_{t-1})}{(1 + \eta_{t-1})} \mathbf{e}_t^i \quad \forall t, i. \quad (54)$$

■

Table 1: Optimal Fiscal Policy

$\lambda$	Capital tax	Labor tax	Net int. rate	Debt/GDP	Capital/GDP
0.93	58.1%	17.9%	3.1%	18.1%	2.2
0.94	49.2%	19.8%	3.2%	12.8%	2.3
0.947	40.8%	21.4%	3.3%	8.7%	2.4
0.95	36.6%	22.1%	3.3%	6.8%	2.5
0.96	17.7%	24.7%	3.4%	-0.3%	2.7
0.97	0.3%	19.3%	3.1%	-30.1%	3.0
0.98	-6.5%	10.4%	2.2%	-91.5%	3.3
ME	35.0%	24.0%	3.6%	24.0%	2.5

Table 2: Sensitivity analysis

$\sigma$	Capital tax	Labor tax	Net int. rate	Debt/GDP
1.0	40.8%	21.8%	3.3%	8.7%
1.5	23.0%	27.0%	4.3%	9.5%
2.0	8.9%	27.9%	5.1%	2.6%
2.5	3.4%	22.2%	5.4%	-9.7%
3.0	-1.2%	17.7%	5.6%	-18.8%
4.0	-6.9%	10.2%	6.0%	-32.2%

Figure I: Capital and labor age-dependent taxes

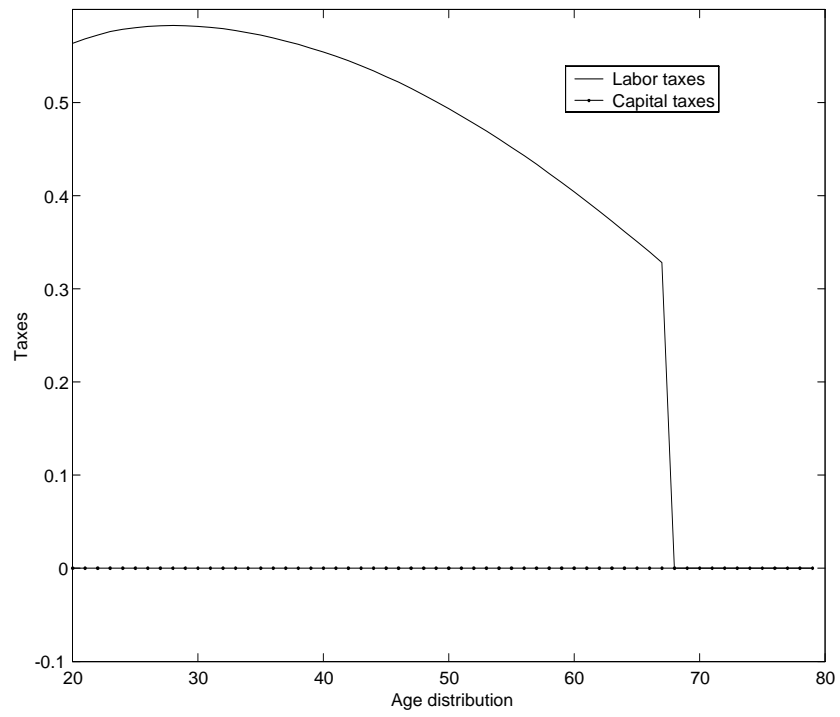
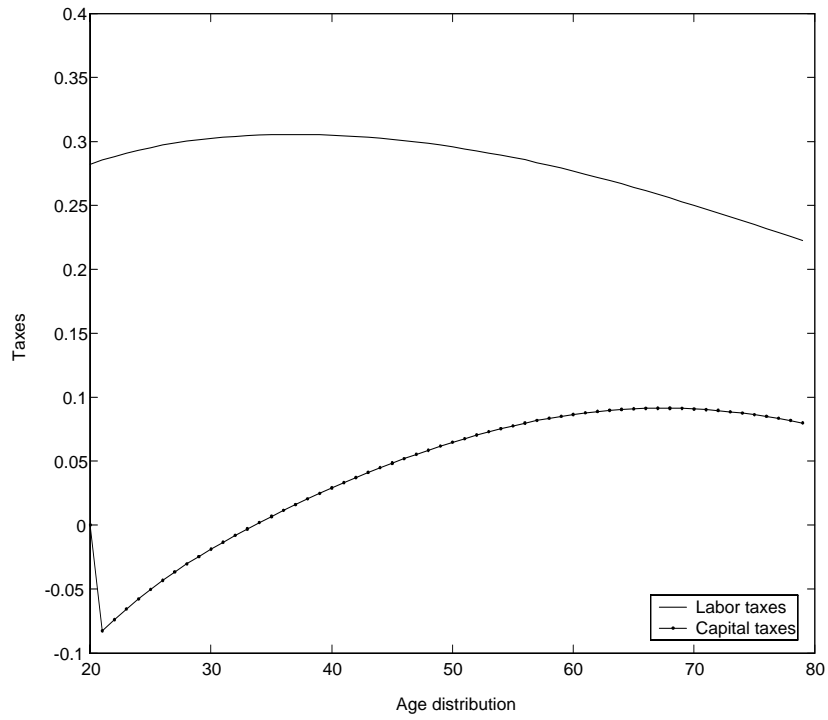


Figure II: Sensitivity analysis  
Case  $\sigma = 2$



Case  $\sigma = 3$

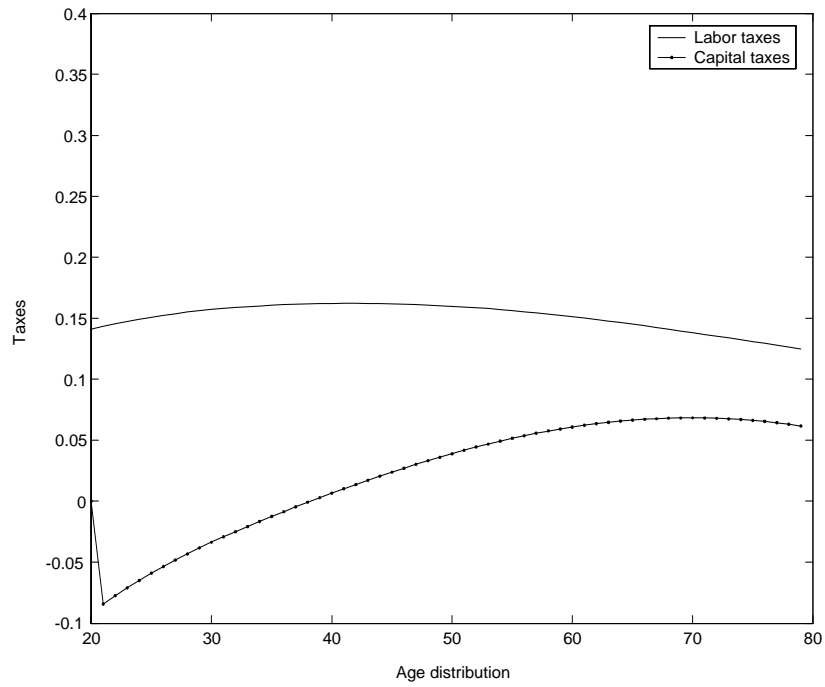


Figure III: Consumption taxes

