

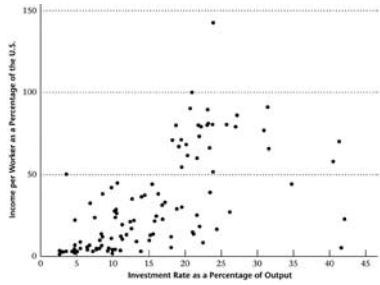
ECONOMIC GROWTH THEORY- What determines the Trend Line?

Economic Growth Facts

- Before the Industrial Revolution(1800), standards of living differed little over time and across countries.
- Since the Industrial Revolution, per capita income growth has been sustained in the richest countries.

- Countries in which a relatively large(small) fraction of output is channeled into investment tend to have relatively high(low) standard of living.

Figure 6.2 Output per Worker vs. Investment Rate



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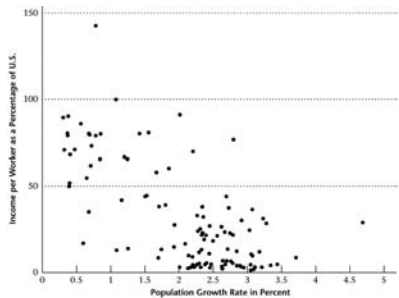
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- There is a negative correlation between the population growth rate and output per worker.

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Figure 6.3 Output per Worker vs. the Population Growth Rate

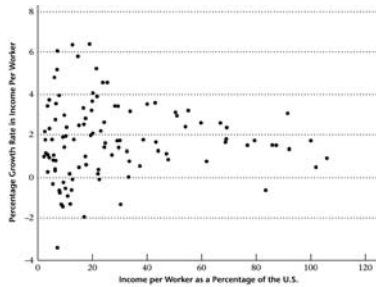


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- Between 1800 and 1950, there was a divergence between living standards in the richest and poorest countries.

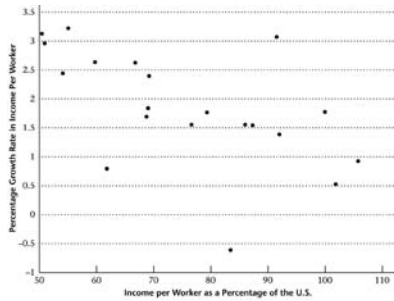
Figure 6.4 No Convergence Among All Countries



- There is essentially no correlation across countries between the level of output per worker in 1960 and the average rate of growth of output per worker for the years 1960-1995. (Convergence Issue)

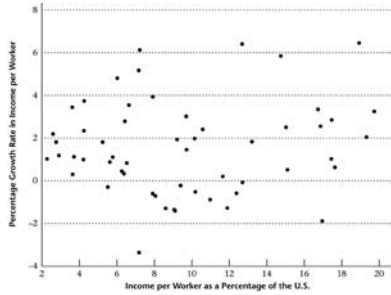
- Among the richest countries, there is a negative correlation between the level of output per worker in 1960 and the average rate of growth in output per worker for the years 1960-1995.

Figure 6.5 Convergence Among the Richest Countries



- Among the poorest countries, there is essentially no correlation between the level of output per worker in 1960 and the average rate of growth in output per worker for the years 1960-1995.

Figure 6.6 No Convergence Among the Poorest Countries



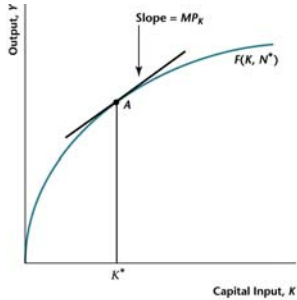
Background on Production Functions

Definition: The production function describes the technological possibilities for converting factor inputs into output

$$Y = zF(K, N^d)$$

DEFINITION: The marginal product of a factor of production is the additional output that can be produced with one additional unit of that factor input, holding everything else constant.

Figure 4.15 Production Function, Fixing the Quantity of Labor and Varying the Quantity of Capital



Properties of the production function

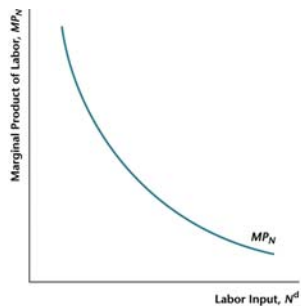
- The production function exhibits constant returns to scale
$$zF(xK, xN) = zF(K, N)$$

- The production function has the property that output increases when either the capital input or the labor input increases – the marginal product of labor and capital are positive

$$MP_K > 0, MP_N > 0$$

- The marginal product of an input decreases as the quantity of that input increases.

Figure 4.16 Marginal Product of Labor Schedule for the Representative Firm



The Malthusian Model of Economic Growth

• Malthusian Theory of Growth

- The view that real GDP growth is temporary and that when real GDP per person rises above the subsistence level, a population explosion eventually brings real GDP per person back to the subsistence level.
- The subsistence **real wage** is the minimum real wage rate is needed to maintain life.

A Simple Malthusian Model

Production The Aggregate Function:

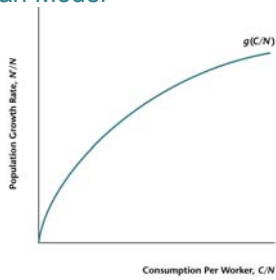
$$Y_t = z_t F(L, N_t)$$

where L is fixed supply of land

Population growth depends on the quantity of consumption per worker.

$$N_{t+1}/N_t = g(C_t/N_t)$$

Figure 6.7 Population Growth Depends on Consumption per Worker in the Malthusian Model



Equilibrium:

$$C_t = Y_t$$

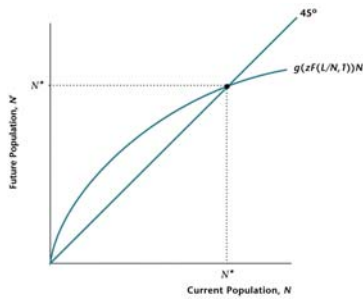
or

$$zF(L, N_t) = Y_t$$

Combine population growth equation
and equilibrium equation

$$\begin{aligned} N_{t+1}/N_t &= g(zF(L, N_t)/N_t) \\ &= g(zF(L/N_t), 1)N_t \end{aligned}$$

Figure 6.8 Determination of the
Population in the Steady State



Given steady state N^* , steady state
consumption is:

$$C^* = zF(L, N^*)$$

Analysis of the Steady State in the Malthusian Model

We should rewrite the model in per capita form.

$$Y_t/N_t = zF(L/N_t, 1)$$

Or

$$y_t = zf(l_t)$$

Where l is land per worker

The per worker equilibrium condition

$$c_t = zf(l_t)$$

The population growth equation:

$$N_{t+1}/N_t = g(c)$$

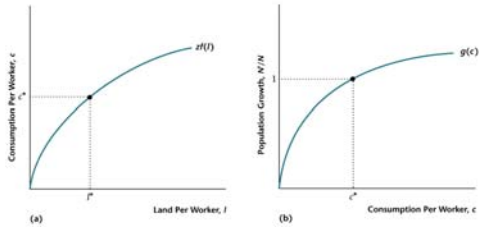
In the steady state:

$$N^* = N_{t+1} = N_t$$

Then,

$$N_{t+1}/N_t = g(c) = 1$$

Figure 6.10 Determination of the Steady State in the Malthusian Model



What happens if total factor productivity increases?

Figure 6.11 The Effect of an Increase in z in the Malthusian Model

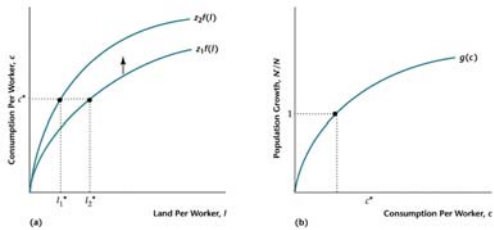


Figure 6.12
Adjustment to the
Steady State in
the Malthusian
Model When z
Increases

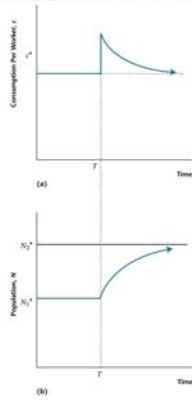
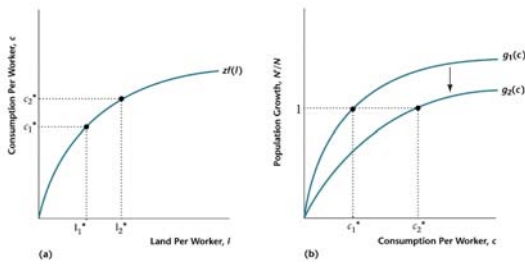


Figure 6.13 Population Control in
the Malthusian Model



Why was Malthus wrong?

- No allowance for the effect of increases in the capital stock on production
- Did not account for all the effects of economic forces on population growth.

The Solow Model of Economic Growth

The Basic Equilibrium Model

- Consumers

$$N_{t+1} = (1+n)N_t \quad n = \text{population growth rate}$$

$$C_t = (1+s)Y_t \quad s = \text{savings rate}$$

- Firms

$$Y_t = z_t F(K_t, N_t)$$

Or

$$y_t = z_t f(k_t)$$

$$K_{t+1} = (1-d)K_t + I_t$$

d = depreciation rate

- Equilibrium Conditions

$$Y_t = C_t + I_t$$

$$N_t = N_t^d$$

Derivation of Fundamental Equation of Growth

$$Y_t = C_t + I_t$$

$$Y_t = (1-s)Y_t + K_{t+1} - (1-d)K_t$$

Solve for K_{t+1}

$$K_{t+1} = szF(K_t, N_t) - (1-d)K_t$$

Divide both sides by N_t

$$\frac{K_{t+1}N_{t+1}}{N_{t+1}N_t} = \frac{szf(k_t) - (1-d)k_t}{N_{t+1}N_t}$$

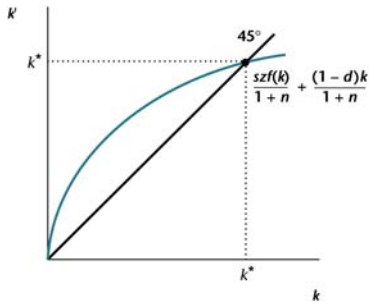
$$N_{t+1}N_t$$

$$k_{t+1}(1+n) = szf(k_t) - (1-d)k_t$$

Or

$$k_{t+1} = \frac{szf(k_t) - (1-d)k_t}{(1+n)}$$

Figure 6.15 Determination of the Steady State Quantity of Capital per Worker



- The Solow model tells us if s , n , and z are constant, then income per worker, y , will be constant in the long run
- All real variables will grow at the rate n .

$$y = Y/N$$

Or

$$Y = y \cdot N$$

$$Y = zf(k^*)N$$

$zf(k^*)$ – is constant in the steady state

N - grows at $(1+n)$

Hence,

Y grows at $(1+n)$

$$C = (1-s)Y$$
$$= (1-s)zf(k^*)N$$

Again, we see that C will grow at $(1+n)$.

- In the long run, the Solow model tells us that growth in key macroeconomic aggregates is determined by exogenous labor force growth, given s and z .

What causes the steady state equilibrium to change?

- The Fundamental Equation of Growth Once Again

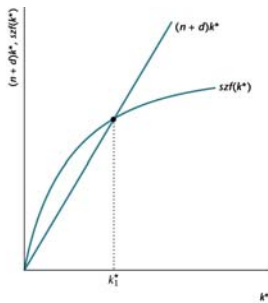
$$k_{t+1} = \frac{szf(k_t)}{(1+n)} - \frac{(1-d)k_t}{(1+n)}$$

Steady State: $k^* = k_{t+1} = k_t$

Hence, steady state occurs at:

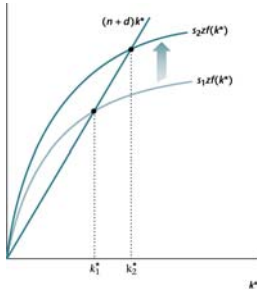
$$szf(k^*) = (n+d)k^*$$

Figure 6.16 Determination of the Steady State Quantity of Capital per Worker



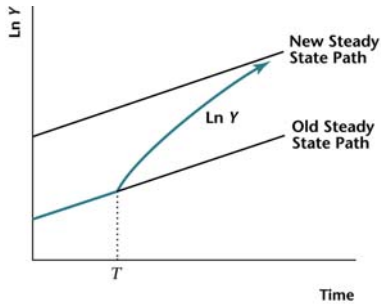
If we increase the savings rate, will we grow faster?

Figure 6.17 Effect of an Increase in the Savings Rate on the Steady State Quantity of Capital per Worker



- If s increases, K/N and Y/N will be greater in the new steady state.
- At the new steady state, K/N and Y/N will be constant
- Thus, once at new steady state, there will be no growth effects.

Figure 6.18 Effect of an Increase in the Savings Rate at Time T



Can a Country have too high a savings rate?

Figure 6.19 Steady State Consumption per Worker

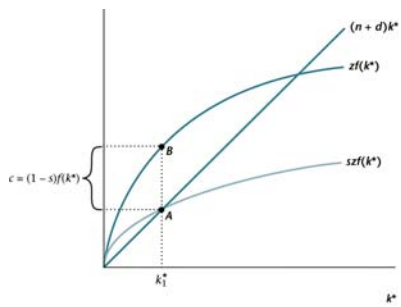
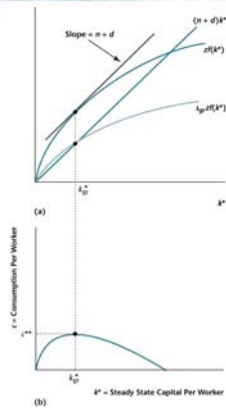
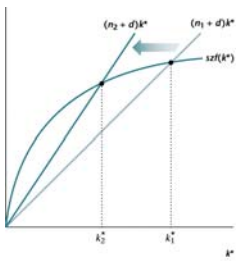


Figure 6.20 The Golden Rule Quantity of Capital per Worker



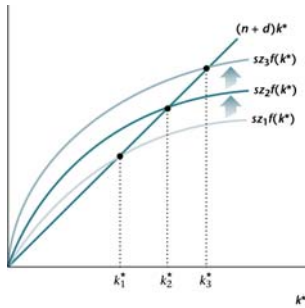
What happens if the population growth rate decreases?

Figure 6.21 Steady State Effects of an Increase in the Labor Force Growth Rate



What happens if TFP increases?

Figure 6.22 Increases in Total Factor Productivity in the Solow Growth Model



Key Insight from Solow Model – an increase in z , and increase in s , or a decrease in n imply a one-time increase in a country’s standard of living. Unbounded growth in the standard of living requires z grows!

Limitations of the Solow Model

- Economic Growth is Exogenous
- The Solow model predicts convergence of standard of livings.

Figure 7.1 Rich and Poor Countries and the Steady State

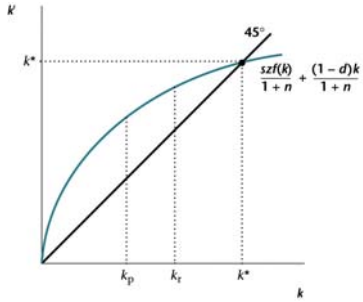
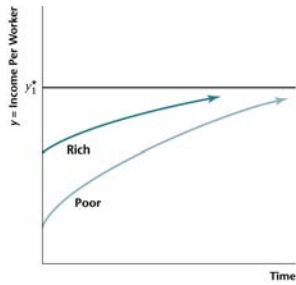


Figure 7.2 Convergence in Income per Worker Across Countries in the Solow Growth Model



Solow Growth Accounting- What really accounts for growth?

How to do Solow Growth Accounting

- Step 1 – Get data on output(Y), capital(K), labor input(N).
- Step 2- Specify labor share of output(64%)
- Step 3- Write down the production function

$$Y = zK^{0.36}N^{0.64}$$

- Step 4 – Calculate the Solow Residual

$$z = Y/(K^{0.36}N^{0.64})$$

Example

Year	Y	K	N
1990	6709.9	20871.1	118.8
1991	6676.4	21207.6	117.7
1992	6880.0	21577.4	118.5
1993	7062.6	22027.7	120.3
1994	7347.7	22530.2	123.1
1995	7543.8	23072.9	124.9
1996	7813.2	23701.0	126.7
1997	8159.5	24383.6	129.6
1998	8508.9	25175.2	131.5
1999	8859.0	26033.2	133.5
2000	9191.4	26933.8	136.9
2001	9214.5	27711.2	136.9

Calculation of Solow residual

$$\begin{aligned}
 1990: z &= Y/(K^{0.36}N^{0.64}) \\
 &= (6709.9)/((20871.1)^{0.36}(118.8)^{0.64}) \\
 &= 8.7861 \\
 2001: z &= Y/(K^{0.36}N^{0.64}) \\
 &= (9214.5)/((27711.2)^{0.36}(136.9)^{0.64}) \\
 &= 9.9498
 \end{aligned}$$

Components in Rate of Change

data	dln(y)	dln(k)	dln(n)	dln(z)
1991	-0.4992	1.6122	-0.9259	-0.4797
1992	3.0495	1.7437	0.6796	1.6721
1993	2.6540	2.0879	1.5189	0.9152
1994	4.0367	2.2812	2.3275	1.6869
1995	2.6688	2.4087	1.4622	0.8515
1996	3.5711	2.7222	1.4411	1.6394
1997	4.4322	2.8800	2.2888	1.8838
1998	4.2821	3.2464	1.4660	2.1338
1999	4.1145	3.4081	1.5209	1.8769
2000	3.7521	3.4594	2.5468	0.8531
2001	0.2513	2.8863	0.0000	-0.7703

How do you determine which factor contributes the most to growth in output?

$$\begin{aligned}
 d\ln(y) &= 0.36d\ln(k) + 0.64d\ln(n) + d\ln(z) \\
 d\ln(y) &\quad d\ln(y) \quad d\ln(y) \quad d\ln(y)
 \end{aligned}$$

Relative Importance of Factors

Date	K	N	z
1991	-1.1625	1.1869	0.9610
1992	0.2058	0.1464	0.6451
1993	0.2831	0.3663	0.3448
1994	0.2034	0.3690	0.4178
1995	0.3249	0.3506	0.3190
1996	0.2744	0.2582	0.4591
1997	0.2339	0.3305	0.4250
1998	0.2729	0.2191	0.4983
1999	0.2982	0.2365	0.4562
2000	0.3319	0.4344	0.2274
2001	4.1344	0.0000	-3.0653

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Table 6.2 Measured GDP, Capital Stock, Employment, and Solow Residual

Year	\hat{Y} (billions of 1996 dollars)	\hat{K} (billions of 1996 dollars)	\hat{N} (millions)	\hat{z}
1950	1686.6	5553.1	58.89	5.574
1960	2376.7	7920.9	65.78	6.439
1970	3578.0	11547.1	78.67	7.548
1980	4900.9	15922.3	99.30	7.934
1990	6707.9	20871.1	118.80	8.784
2000	9191.4	26993.8	136.90	10.027

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How do you calculate the average annual growth rate in a variable?

$$g_{m,n} = (X_n/X_m)^{1/(n-m)} - 1$$

$$m = 1950$$

$$n = 1960$$

$$g_{m,n} = (2376.7/1686.6)^{1/(10)} - 1$$

$$= 0.0349 \text{ (3.49\%)}$$

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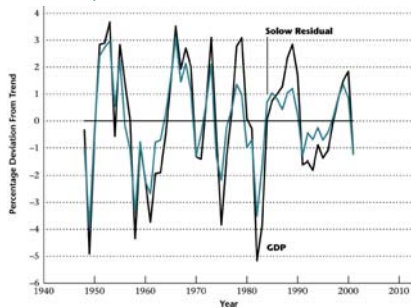
Table 6.1 Average Annual Growth Rates in the Solow Residual

Years	Average Annual Growth Rate
1950–1960	1.45
1960–1970	1.60
1970–1980	0.50
1980–1990	1.02
1990–2000	1.33

Table 6.3 Average Annual Growth Rates

Years	\hat{Y}	\hat{K}	\hat{N}	\hat{z}
1950–1960	3.49	3.62	1.11	1.45
1960–1970	4.18	3.84	1.81	1.60
1970–1980	3.20	3.27	2.36	0.50
1980–1990	3.19	2.74	1.81	1.02
1990–2000	3.20	2.61	1.43	1.33

Figure 6.24 Percentage Deviations from Trend in Real GDP (black line) and the Solow Residual (colored line), 1948–2001



What can we learn from the East Asian Countries?

Table 6.4 East Asian Growth Miracles
(Average Annual Growth Rates)

	Output	Capital	Labor	Total Factor Productivity
Hong Kong (1966–1991)	7.3%	7.7%	2.6%	2.3%
Singapore (1966–1990)	8.7%	10.8%	4.5%	0.2%
South Korea (1966–1990)*	10.3%	12.9%	5.4%	1.7%
Taiwan (1966–1990)*	9.4%	11.8%	4.6%	2.6%
United States (1966–1990)	3.0%	3.2%	2.0%	0.6%
