

# THE BASIC TWO-PERIOD MODEL: THE FISHERIAN APPROACH

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## Goals

- We operate in a dynamic world.
- How do you make decisions in such an environment?
- We will study this question in terms of a two period model
- What are the implications of this dynamic model for how consumers respond to government policy changes?

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## A TWO-PERIOD MODEL OF THE ECONOMY

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## Why study a two period model when individuals live more than two periods?

- Two period models are simple to analyze
- Two period models capture the essential of dynamic decision making by consumers and firms.

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## Some Simplifying Assumptions

- Income is exogenous- $(y_1, y_2)$
- No work-leisure decision
- Taxes are lump sum(t)

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- Private bonds(b) are the only asset available for savings

if you want to save you buy a bond

$$b > 0$$

if you want to borrow, you sell a bond

$$b < 0$$

- A bond is a promise to pay  $1+r$  units of the consumption good in the future period.

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## The Consumer's Budget Constraints (Hint: What are the sources and uses of funds?)

- Current Period(1)

$$c_1 + b_2 = y_1 - t_1$$

(Why doesn't bond income appear on RHS?)

- Future Period(2)

$$c_2 = y_2 - t_2 + (1+r)b_2$$

(Why don't you buy(sell) bonds in period2?)

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## Derivation of the Lifetime Budget Constraint

- Step 1- Rewrite period 1 budget constraint in terms of  $b_2$ .

$$b_2 = y_1 - t_1 - c_1$$

- Step 2-Substitute this equation into the second period budget constraint.

$$\begin{aligned} c_2 &= y_2 - t_2 + (1+r)b_2 \\ &= y_2 - t_2 + (1+r)[y_1 - t_1 - c_1] \end{aligned}$$

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- Step 3 – Rewrite the equation and divide both side of the equation by  $(1+r)$ .

$$c_1 + (1+r)^{-1}c_2 = y_1 + (1+r)^{-1}y_2$$

or

$$c_1 + (1+r)^{-1}c_2 = we$$

$$we = y_1 + (1+r)^{-1}y_2$$

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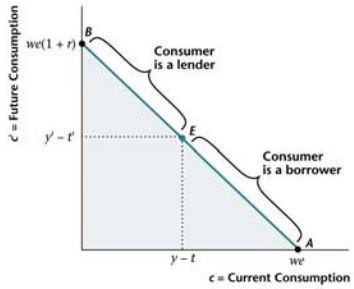
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## Figure 8.1 Consumer's Lifetime Budget Constraint



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## Consumer Preferences

- $U = U(c_1, c_2)$

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## Properties of the Utility Function

- More is always preferred to less.
- The consumer likes diversity in the consumption bundle
- Current and Future consumption are normal goods.

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## Slope of the indifference curve

- Definition: The slope of the indifference curve is minus the marginal rate of substitution between current period consumption and future period consumption.

$$\text{slope} = -MRS_{c_1, c_2} = -MU_{c_2} / MU_{c_1}$$

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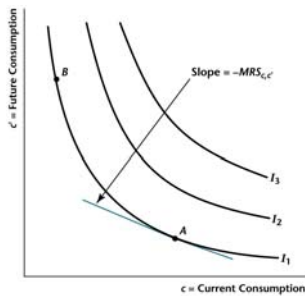
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**Figure 8.2** A Consumer's Indifference Curves



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## Consumer Optimization

- The Consumer Problem:

$$\text{Maximize } U = U(c_1, c_2)$$

Subject to

$$c_1 + (1+r)^{-1}c_2 = y_1 + (1+r)^{-1}y_2$$

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**Solution: Find the point where the indifference curve is tangent to the budget constraint.**

$$MRS_{c_1, c_2} = 1+r$$

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**The consumer optimizes by choosing the consumption bundle on the budget constraint where the rate at which you are willing to trade off current consumption for future consumption is the same as the rate at which you can trade current consumption for future consumption.**

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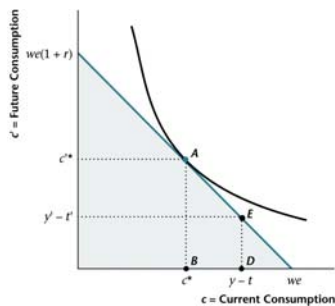
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**Figure 8.3 A Consumer Who Is a Lender**




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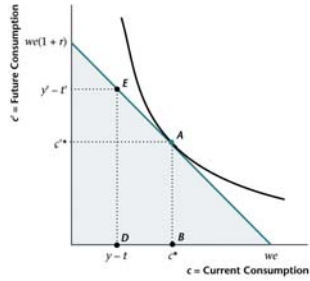
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**Figure 8.4** A Consumer Who Is a Borrower




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**The Notion of Consumption Smoothing-** attempt to keep consumption similar each period.

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**What happens if current period income increases?**

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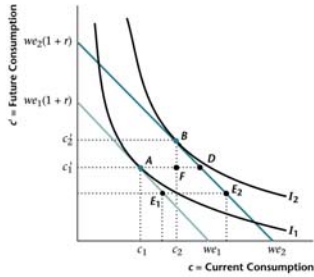
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**Figure 8.5** The Effects of an Increase in Current Income for a Lender




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**Conclusion: The increase in current income results in**

- An increase in consumption in both periods((smoothing)
- Increase in savings

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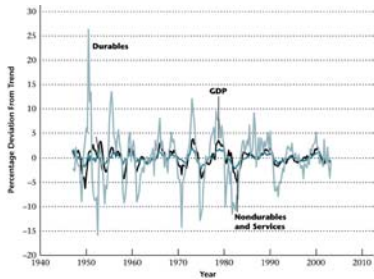
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**Figure 8.6** Percentage Deviations from Trend in GDP and Consumption, 1947–2003




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Data indicates that consumption is smoother than income, as the theory predicts. However, consumption does not appear to be as smooth as theory predicts. Why?

- Imperfections in the capital market
- Does not take into account the interactions of consumers

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Does it matter if we replace an increase in current income with an increase in future income?

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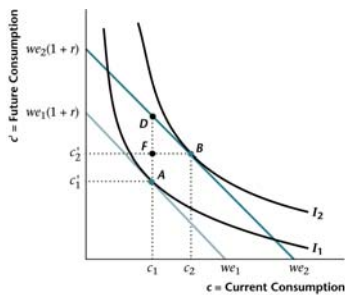
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Figure 8.7 An Increase in Future Income




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## An increase in future income leads to smoothing backward.

- Current consumption increases
- Savings decrease
- Future consumption increases, but less than the increase in future income.

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## The Bush tax cut is temporary. Does it matter whether an increase in income is permanent or temporary?

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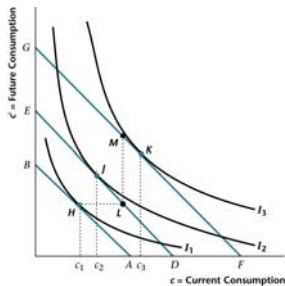
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## Figure 8.8 Temporary Versus Permanent Increases in Income



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## Findings:

- A temporary increase in income, (only the current period), results in a small change in consumption today and an increase in savings.
- A permanent increase in income, (both periods), results in an big increase in income each period, and little change in savings.

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## What happens if interest rates increase? (Substitution Effects)

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## How is the budget constraint impacted?

- $1/(1+r)$  is the price of future consumption relative to current consumption
- An increase in  $r$  causes  $c_2$  to be relative cheaper than  $c_1$ .
- An increase in  $r$  cause the budget constraint to rotate clockwise about the endowment point

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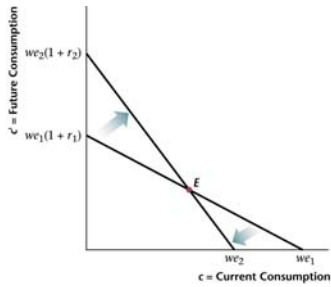
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**Figure 8.11** An Increase in the Real Interest Rate




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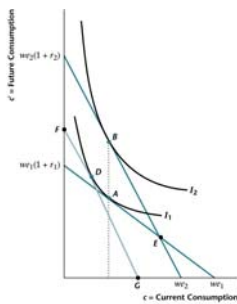
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**Figure 8.12** An Increase in the Real Interest Rate for a Lender




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**Income and Substitution Effects for a Lender**

Effect	$C_1$	$C_2$	S
Income	+	+	
Substitution	-	+	
Total	?	+	?

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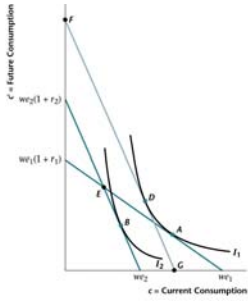
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**Figure 8.13** An Increase in the Real Interest Rate for a Borrower




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**Income and Substitution Effects for a Borrower**

Effect	$C_1$	$C_2$	S
Income	-	+	
Substitution	-	-	
Total	-	?	+

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**The Consumption Demand Curve**

- We can derive the implicit consumption demand curve from the consumer's optimization problem.
- Assume the substitution effect dominates the income effect when the consumer is a lender, (i.e., an increase in  $r$  causes  $c_1$  to fall)

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- Ask. When  $y_1$  increases, what happens to  $c_1$ ?
- The slope of the  $c_1$  is the marginal propensity to consume.

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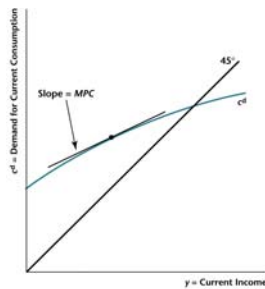
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**Figure 8.15** A Consumer's Demand for Current Consumption Goods,  $c^d$ , as a Function of Current Income




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**What happen to the  $c_1$  curve if  $y_2$  increases?**

- Assume the substitution effect dominates the income effect.
- The  $c_1$  curve shifts upward

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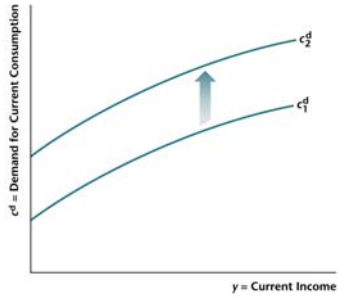
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**Figure 8.16** A Shift in a Consumer's Demand for Current Consumption




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**Application:** What happens to current consumption if the borrower is constrained from entering the credit market?

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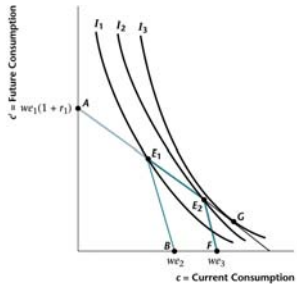
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**Figure 8.22** Effects of a Tax Cut for a Consumer with Different Borrowing and Lending Rates




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## What do we learn?

- The borrowing constrained household increases current consumption a lot and decreases future consumption when income in period 1 increases.
- If a lot of poor household are credit constrained, this may explain the excess volatility of consumption to income changes.

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## THE GOVERNMENT SECTOR

- The government spends each period.
- The government can tax household each period or issue government bonds.
- Government bonds are perfect substitutes for private bonds

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## Derivation of Government Budget Constraint

- Period 1 constraint:  
$$G_1 = T_1 + B_2$$
- Period 2 constraint:  
$$G_2 + (1+r)B_2 = T_2$$

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- Government present value budget constraint

Divide both sides of the second period constraint by  $(1+r)$

Add first and second period constraints together.

$$G_1 + G_2/(1+r) = T_1 + T_2/(1+r)$$

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## Competitive Equilibrium in the Two Period Model

- Given  $r$ , each consumer chooses first- and second-period consumption and savings optimally.
- The government present value budget constraint holds

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- The Credit Market clears

$$S^p = B$$

$S^p$  = aggregate quantity of private savings

$B$  = Debt issued by the government

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## The credit market equilibrium condition implies goods market clearing

Private and government savings are

$$S^p = Y_1 - C_1 - T_1$$

$$B = G_1 - T_1$$

Credit market equilibrium is:

$$S^p = B$$

$$Y_1 - C_1 - T_1 = G_1 - T_1$$

Or

$$Y_1 = C_1 + G_1$$

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