

AN INTERTEMPORAL MODEL WITH INVESTMENT

Goals

- Add Investment Depending to the Model
- Show how the model can be employed to predict how variables respond to exogenous events.

The Representative Consumer

- Supplies labor to firms in the current and future period
- Consumes goods produced by firms
- Receives profits of the firm
- Pays taxes to the government

Representative Consumer Problem

$$\max U(c_1, c_2, l_1, l_2)$$

Subject to:

$$c_1 + s_1 = w_1(h_1 - l_1) + \pi_1 - T_1$$

$$c_2 = w_2(h_2 - l_2) + \pi_2 - T_2 + (1 + r_2)s_2$$

Or

$$c_1 + c_2 / (1 + r_2) = w_1(h_1 - l_1) + \pi_1 - T_1 + (w_2(h_2 - l_2) + \pi_2 - T_2) / (1 + r_2)$$

Review of conditions that determine how the consumer should behave

- Current and Future work-leisure decision

$$MU_l / MU_c = MRS_{l,c} = w$$

- Consumption-savings decision

$$MU_{c_1} / MU_{c_2} = MRS_{c_1, c_2} = (1 + r)$$

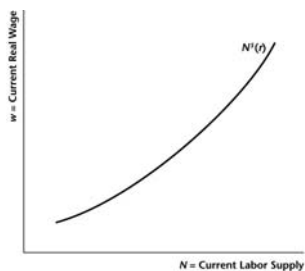
Determinants of Labor Supply

- If the substitution effect dominates the income effect, an increase in w results in more labor supply
- If r increases, labor supply increases to take advantage of higher return on savings
- If wealth increases, labor supply decreases.

Derivation of the Labor Supply Curve

- The labor supply curve is a positive relationship between the current wage and hours supplied.

Figure 9.1 The Representative Consumer's Current Labor Supply Curve



What causes the labor supply curve to shift?

- A change in the interest rate
- A change in wealth

Figure 9.2 An Increase in the Real Interest Rate Shifts the Current Labor Supply Curve to the Right

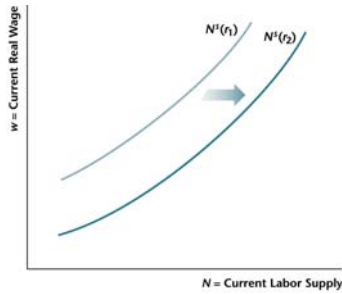
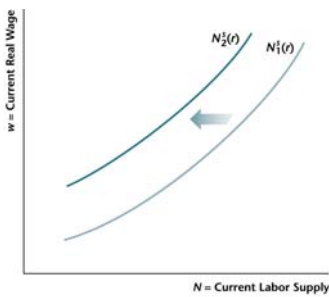


Figure 9.3 Effects of an Increase in Lifetime Wealth



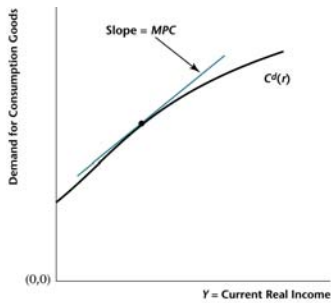
Determinants of Consumption Demand

- The real interest rate
- Lifetime wealth which depends on current income, future income, and the present value of taxes

Derivation of the Consumption Demand Curve

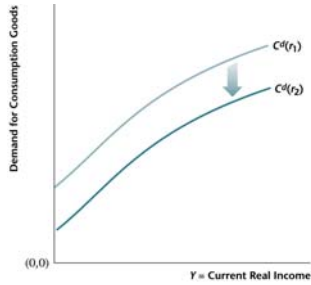
- The consumption demand curve represents a positive relationship between current real income and consumption demand.

Figure 9.4 The Representative Consumer's Current Demand for Consumption Goods Increases with Income



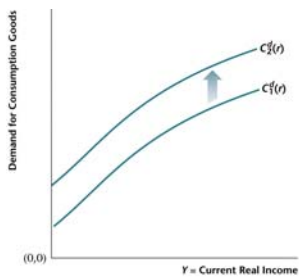
- An increase in the real interest rate means you save more at each income level, so the consumption demand curve shifts down.

Figure 9.5 An Increase in the Real Interest Rate from r_1 to r_2 Shifts the Demand for Consumption Goods Down



- An increase in lifetime wealth means the consumption demand curve shifts upward at each current income level.

Figure 9.6 An Increase in Lifetime Wealth for the Consumer Shifts Up the Demand for Consumption Goods.



The Representative Firm

- Produces goods using labor and capital.
- Makes investment decisions
- Hires labor from consumers
- Profits are returned to the consumer

The Production Decision

- In Period 1
 $Y_1 = z_1(K_1, h_1)$
- In Period 2
 $Y_2 = z_2(K_2, h_2)$
- The Capital Evolution equation
 $K_2 = (1-d)K_1 + I_1$

What does the capital evolution equation tell us?

- In period 1, the firm must decide what size capital stock it wants in period 2 and make the appropriate investment decision.
- Using current production for investment purposes, means less goods can be sold to the consumer.

- Since the firm lives only two periods, the capital left at the end of period 2 is sold and converted to consumption goods, one for one.

The Value of the Firm

- The value of any firm is the present value of profits.

$$V = \frac{\pi_1 + \pi_2}{1+r}$$

- Period 1 profits

$$\pi_1 = Y_1 - w_1 h_1 - l_1$$

- Period 2 profits

$$\pi_2 = Y_2 - w_2 h_2 + (1-d)K_2$$

The Representative Firm's Investment Decision

What is the cost of an in additional unit of investment?

- $MC(I) = 1$
(The cost of an additional unit of investment is the value of sales of consumption goods you lose)

What is the benefit from another unit of investment goods?

- An additional investment good increases capital next period. The additional output from the increase in capital is MP_K .
- The capital stock at the end of the second period is exchanged for consumption goods which can sold.
- Since these benefits occur next period, they must be discounted.

$$MB(I) = [MP_{K2} + 1 - d] / (1 + r)$$

The Optimal Investment Rule

$$MB(I) = [MP_{K2} + 1 - d] / (1 + r) = MC(I) = 1$$

Or

$$[MP_{K2} + 1 - d] / (1 + r) = 1$$

$$[MP_{K2} + 1 - d] = (1 + r)$$

Or

$$MP_{K2} - d = r$$

Summary

The opportunity cost of investing in more capital is the real rate of interest, which is the rate of return on the other asset(Bonds) in this economy.

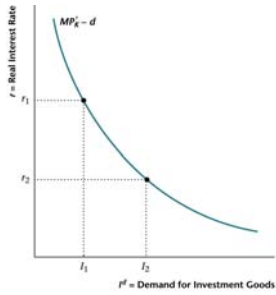
What are the determinants of investment?

- The real interest rate
- The depreciation rate
- Anything that changes the marginal productivity of capital next period
 - Productivity in the second period
 - The capital stock
 - Labor

Derivation of the Investment Demand Curve

- The investment demand curve shows a downward sloping relationship between the real interest rate and investment demand.

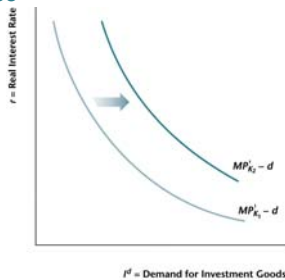
Figure 9.9 Optimal Investment Schedule for the Representative Firm



What shifts the investment demand curve to the right?

- An increase in z_2 increase that periods marginal productivity of capital so I increases.
- If K decreases, the MP_{K_2} increases and investment increases
- A increase in the depreciation rate.

Figure 9.10 The Optimal Investment Schedule Shifts to the Right if Current Capital Decreases or Future Total Factor Productivity Is Expected to Increase



What determines the firm's demand for labor?

- Firms hire labor up until the point where the the marginal productivity of labor is equal to the real wage(w).
- $MP_h = w$

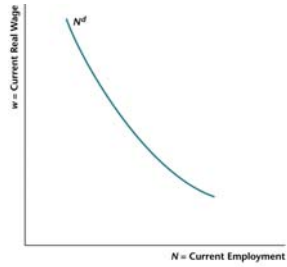
What are the determinants of labor demand?

- The real wage
- Anything that changes the marginal productivity of labor in period 1(z, K, etc.)

Derivation of the Labor Demand Curve

- The labor demand curve is an inverse relationship between the real wage and the quantity of labor demanded.

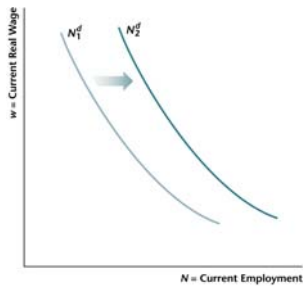
Figure 9.7 The Demand Curve for Current Labor Is the Representative Firm's Marginal Product of Labor Schedule



What causes the labor demand curve to shift?

- A change in z
- A change in K

Figure 9.8 The Current Demand Curve for Labor Shifts Due to Changes in Current Total Factor Productivity z and in the Current Capital Stock K



The Government

- $G_1 + G_2 / (1+r) = T_1 + T_2 / (1+r)$

Determination of the Competitive Equilibrium

- Labor Market
- Current Period Output Market
- Bond Market

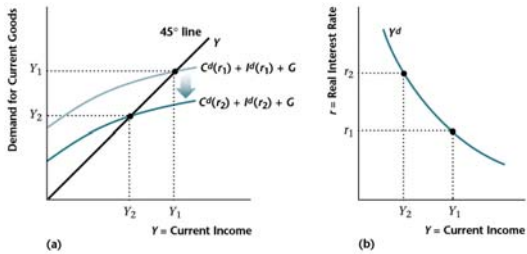
Derivation of the Output Demand Curve

The Output Demand Curve

- The output demand curve is in inverse relationship between output demand and the real interest rate.

Construction of the Output Demand Curve

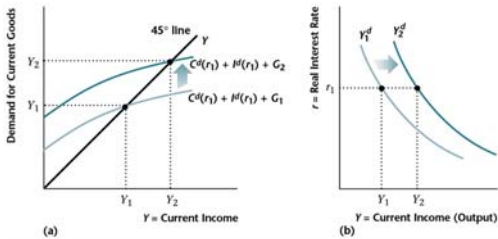
Figure 9.16 Construction of the Output Demand Curve



What causes the output demand curve to shift?

- Anything that causes consumption demand to shift (current income, future income, the present value of taxes)
- Anything that causes investment demand to shift.
- A change in current government spending

Figure 9.17 The Output Demand Curve Shifts to the Right if Current Government Spending Increases



The Supply Curve for Output

- The output supply curve shows the relationship between the real interest rate and the quantity of output supplied on the goods market.

Derivation of the Output Supply Curve

- Use the Labor market diagram
- Use the Production Function Diagram

Figure 9.11
Determination of Equilibrium in the Labor Market Given the Real Interest Rate r

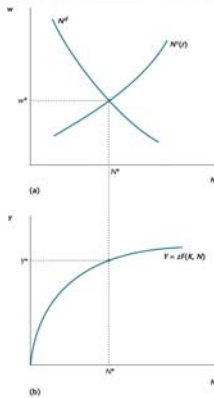
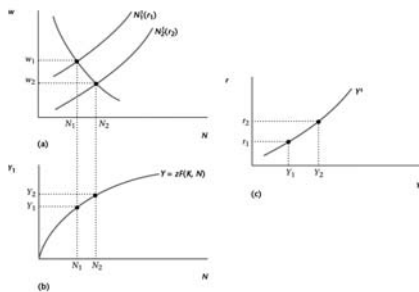


Figure 9.12 Construction of the Output Supply Curve



What cause the output supply curve to shift?

- A change in z_1
- A Change in K
- A Change in labor demand
- A Change in labor supply
- A Change in government Spending(changes the consumers lifetime wealth)

Figure 9.14 An Increase in Current Total Factor Productivity Shifts the Y^s Curve

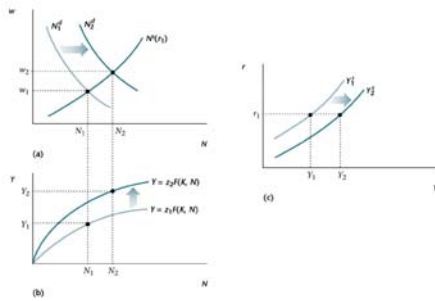


Figure 9.13 An Increase in Current or Future Government Spending Shifts the Y^s Curve

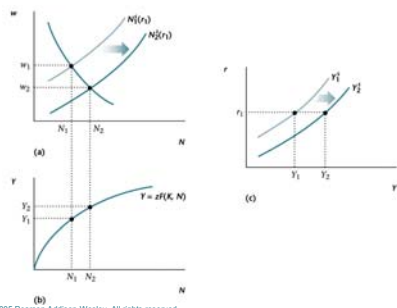
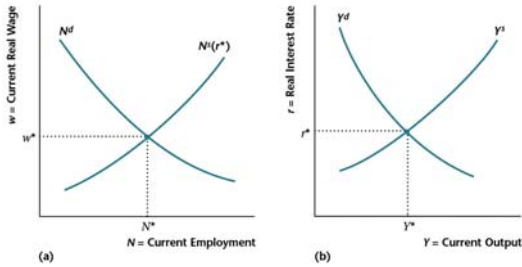


Figure 9.18 The Complete Real Intertemporal Model



Determination of the Competitive Equilibrium and Uses of the Framework

How to use the model to determine the impact of a change in an exogenous variable

- Step 1-Find the equilibrium before any change.
- Step 2-Identify which exogenous variable changes and shift curves that depend on that variable.
- Step 3-Find the new market clearing real interest rate.

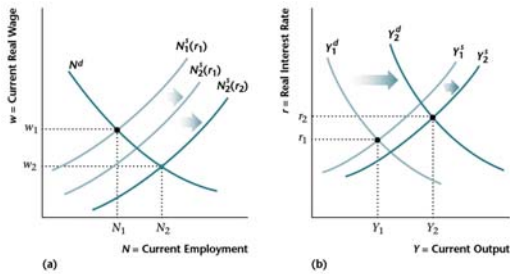
- Step 4- Shift all curve that depend on the real interest rate
- Step 5-Find the new equilibrium and report how critical endogenous variables change. (This gives you your forecast)

Application: A Temporary increase in G in the current period.

- Step 1- Draw the economy in the initial equilibrium. The wage rate is w_1 , the real interest rate is r_1 , and output is Y_1 .
- Step 2- Increase current government spending. This will cause Y^d , $N^s(r_1)$, and Y^s to shift.
- Step 3 – The new equilibrium real interest rate increases to r_2 and output increases to Y_2 .

- Step 4- The increase in r causes labor supply to increase.
- Step 5- Determine the impact on $r, w, N(\text{or } h), Y, G, C, I$.

Figure 9.19 A Temporary Increase in Government Purchases



What are the costs to the temporary increase in G ?

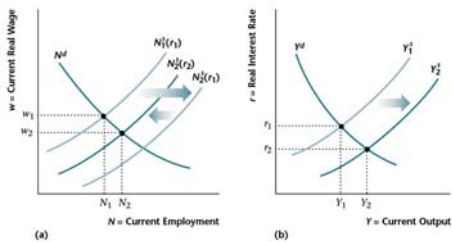
- Lower I so future capacity will be lower.
- Less leisure time

Application: What happens if there is a permanent increase in government spending?

- Step 1 – determine these equilibrium before the change.
- Step 2- The increase in G causes the output demand curve to shift out. This shift is offset by a drop in consumption by the loss in wealth. The demand effect will be small. At the initial r , labor supply and output shift to right.

- Step 3- Shift curves that are impacted by drop in r . The labor supply curve shifts back.
- Step 4 – Find the new equilibrium

Figure 9.20 A Permanent Increase in Government Purchases

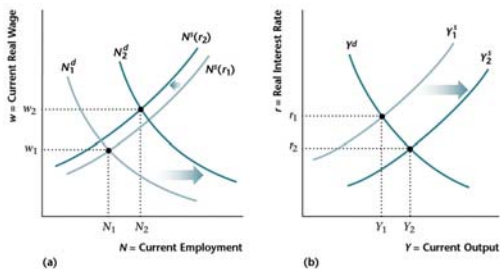


What do we learn?

- The output demand effect is larger when the G change is temporary.
- The output supply effect is larger when the G change is permanent.
- A permanent increase in G does not have the same crowding-out effects.

Application: What happens if current z increases?

Figure 9.23 The Equilibrium Effects of an Increase in Current Total Factor Productivity

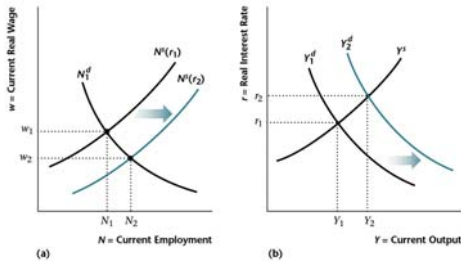


Result:

- A temporary increase in productivity increases Y , I , C , $N(h)$, and w .
- Is this consistent with the data?

Application: What happens if you expect a positive productivity shock next period?

Figure 9.24 The Equilibrium Effects of an Increase in Future Total Factor Productivity



Finding:

Finding:

- A future productivity effect impacts the demand curve rather than the supply curve.
