A MONETARY INTERTEMPORAL MODEL

Goals

- We want to introduce money into the model so we can talk about monetary policy.
- We want to know how an expansionary monetary policy impacts real variables.

The Functions of Money

- Medium of exchange-money can be traded for other goods so the **double coincidence of wants.**
- Store of value – allows trading of current goods for future goods.
- Unit of account – all prices and contracts are denominated in terms of money.

Some background

- We will assume that goods must be purchased with money. This is called a **cash-in-advance** model.
- The introduction of money means distinguish between nominal and real variables.

The Fisher Relationship

- A nominal bond payoff 1+R where R is the nominal interest rate.
- The inflation rate is i:

  $i = (P_2-P_1)/P_1$
- The Fisher relationship:

  $(1+r) = (1+R)/(1+i)$

Where does the Fisher relationship come from?

- If I want to buy a bond today that costs a \$1, I must give up \$1. In real terms, the cost is $\$1/P_1$.
- In the next period, the nominal bond paysoff 1+R. In real terms, the payoff is $(1+R)/P_2$.

- Hence,

  $(1+r) = [1+R]/(1+i)$

  $= [(1+R)/P_2]/(1/P_1)$

  $=(1+R)/(P_2/P_1)$

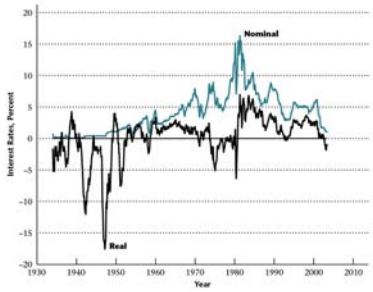
  $=[1+R]/(1+i)$

  Then,

  $(1+R) = (1+r)(1+i)=1 +r +ri+I$

  $R = r+i$

Figure 10.1 Real and Nominal Interest Rates, 1948–2003



How does the introduction of money modify the consumer's problem?

- The consumer must decide how much money to have at the beginning of period t in the prior period.
- The nominal bond position at the beginning of period t is also determined in the prior period.
- At the beginning of the period, the consumer must pay nominal taxes of $P_1 T_1$.

- The cash-in-advance constraint:
  $P_1C_1 = M_1 + B_1(1+r) - P_1T_1 - B^d_{t+1}$
- After the money and bond decisions are made, the consumer makes the consumption choices
- After the consumption choices, the agent is paid for work and profits are distributed.

- The consumer's budget constraint for the nominal economy.

$$P_1C_1 + B^d_2 + M^d_2 = M_1 + B_1(1+R_1) + P_1w_1(h-l_1) + P_1\pi_1 - P_1T_1$$

The Money Demand Function

- Consumers want to hold cash so they can buy goods. The amount of goods purchased will depend on Y(lifetime wealth).
- Consumers lose interest(nominal) payments when they hold money.

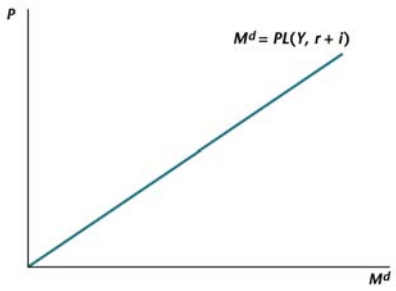
$M^d/P = L(Y,R) = L(Y,r+i)$

This tells us that the demand for money in real terms depends on real wealth, the real interest rate, and the expected inflation rate.

Or,

$M^d = PL(Y,r+i)$

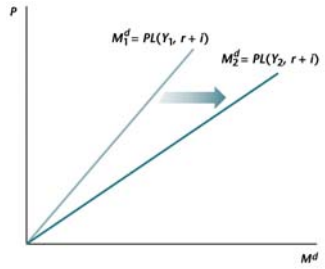
Figure 10.3 The Nominal Money Demand Curve in the Monetary Intertemporal Model



What factors cause the money demand function to shift?

- A change in real income
- A change in the real interest rate
- A change in the expected inflation rate

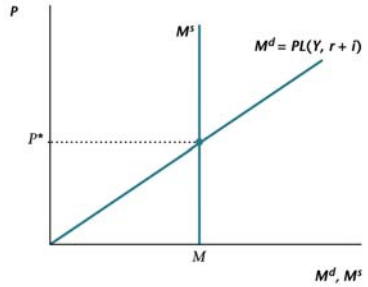
Figure 10.4 The Effect of an Increase in Current Real Income on the Nominal Money Demand Curve



Money Market Equilibrium

- The money supply is determined by FED actions, and thus is exogenous.
- Money demand is determined by the price level and the money demand function.
- Equilibrium requires that money demand equal money supply

Figure 10.5 The Current Money Market in the Monetary Intertemporal Model



Solution of the Intertemporal Monetary Model

- Solution Strategy

  Step 1: Find equilibrium in labor, goods and money market prior to any change.

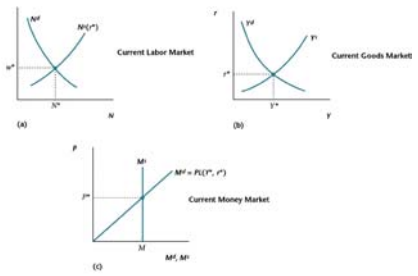
  Step 2: Identify the exogenous variable change and shift appropriate curves.

  Step 3: Find new equilibrium real interest rate

Step 4: Shift curves that depend on the real interest rate.

Step 5: Find new equilibrium and summarize how endogenous variables change.

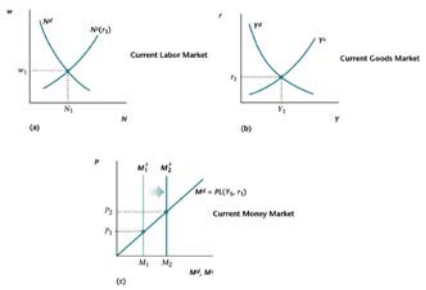
Figure 10.6 The Complete Monetary Intertemporal Model



Application: What happens if the money supply increases?

- Step 1: Show initial equilibrium in the goods, labor, and money markets.
- Step 2: Increase the money supply. This shifts the M^s curve.
- Step 3: The real interest rate doesn't change.
- Step 4: No curves shift due to a change in the real rate.

Figure 10.8 The Effects of a Level Increase in M—The Neutrality of Money



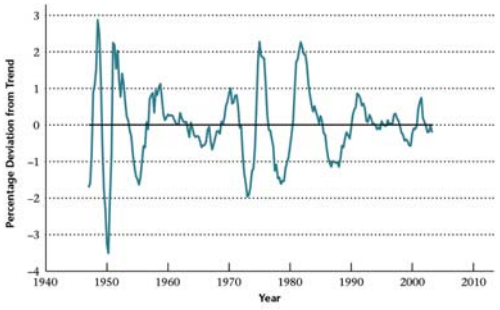
- Step 5:
  output – no change
  consumption – no change
  investment – no change
  real interest rate – no change
  employment – no change
  real wage – no change
  price level - increase

Comment on the result

- An increase in the money supply only affects nominal variables and not real variables.
- This is called the **Classical Dichotomy**.
- In this case, monetary policy is **neutral**.

Application: A temporary decrease in TFP(caused by an increase in oil prices)

Figure 10.11 Percentage Deviations from Trend in the Price Level



- Step 1: Find initial equilibrium
- Step 2: The decrease in z causes the labor demand curve to shift back, and the output supply curve to shift left.
- Step 3: The real interest rate rises.
- Step 4: The rise in the real interest rate causes the labor supply curve to shift to the right. (Why doesn't output demand shift?)

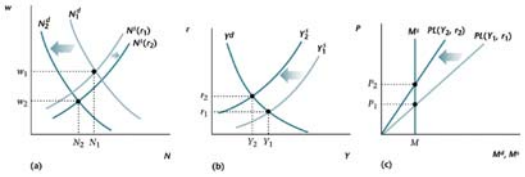
- Step 5: The fall in r and Y causes the money demand function to shift left. Thus, prices fall.
- Step 6: Results:

  Y – falls

  C,I,r,w –fall (procycilcal)

  P – increases(countercyclical)

Figure 10.9 Short-Run Analysis of a Temporary Decrease in Total Factor Productivity



Other factors that shift the money demand function.

- A change in the costs of using other assets as a means of payment.(debit cards)
- A change in the costs of converting other financial assets into money.
- A change in Government Regulations.
- A change in inflation risk.
- A change in the riskiness of other assets.

Example: Suppose the holding of foreign currencies become riskier.

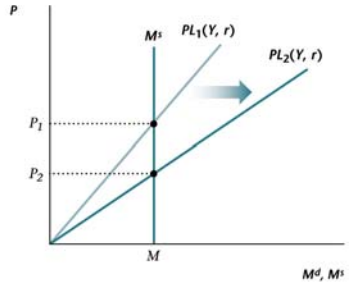
A simple way to see what will happen is to work with the money market equilibrium condition.

$M^s = PL(Y,r\text{-}i)$

Or,

$P = M^s/L(Y,r\text{-}I)$

Figure 10.12 A Shift in the Demand for Money

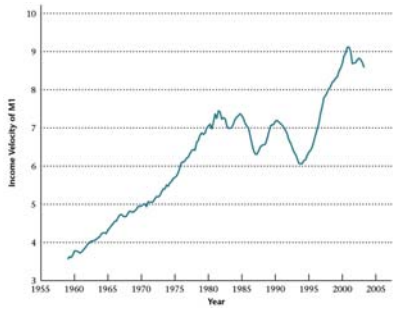


Shifts in Money Demand have implications for the Velocity of money.

- The velocity of money is the ratio of nominal income to the nominal quantity of money.
- $V = PY/M^S = PY/PL(Y,r-i)$

  $= Y/L(Y,r-i)$

Figure 10.14 Velocity of M1



The Quantity Theory of Money

- The quantity theory can be summarized as:

  $M = (1/V)PY$

  where V is fixed.
- Monetarism modifies the quantity theory by stating that velocity is a predictable relationship.

  $M = (1/V(Y,r))PY$

Velocity Changes and Monetary Policy

- Fed follows a policy rule called a Taylor Rule.

  $M^s = M(Y,P)$
- What happens if the Fed follows a price level rule and velocity is unstable?

Figure 10.16 Central Bank Response Stabilizes Price Level

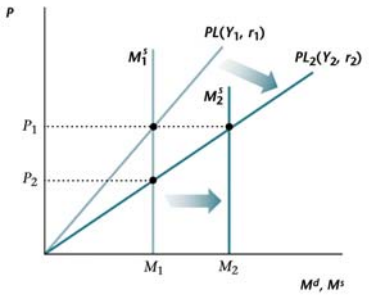
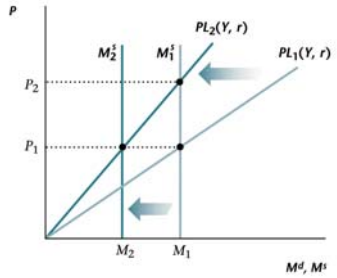


Figure 10.17 Central Bank Does Not Observe the Price Level Response to a Shift in Demand for Money



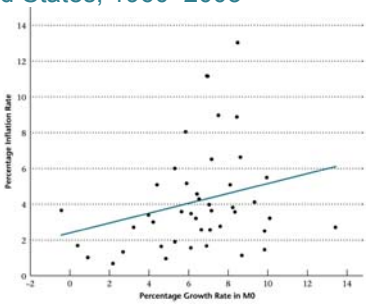
Summary:

- Stabilizing the money supply without somehow accounting for the potential shift in the money demand function does not give the desired result of a stable price level.

Long-Run Inflation in the Monetary Intertemporal Model

- Money is neutral in the long run in that a change in the money supply has no long run real effects.
- Changes in the growth rate of the money supply are not neutral.
- What should be optimal policy?

Figure 15.4 Scatter Plot of the Inflation Rate vs. the Growth Rate in M0 for the United States, 1960–2003



Effects of Long-Run Inflation

- The money supply grows each period by the amount x.

$$M^s_{t+1} = (1+x)\, M^s_t$$

- Look at the money market equilibrium conditions

$$M^s_t = P_t L(Y_t, r_t + i_t)$$

$$M^s_{t+1} = P_{t+1} L(Y_{t+1}, r_{t+1} + i_{t+1})$$

- Take ratio of money market conditions

$$M^s_{t+1} / M^s_t = P_{t+1} L(Y_{t+1}, r_{t+1} + i_{t+1}) / P_t L(Y_t, r_t + i_t)$$

- If real variables do not change, then

$$(1+x) = M^s_{t+1} / M^s_t = P_{t+1} / P_t = (1+i)$$

Or,

$$x = i$$

- This is known as superneutrality- the rate of growth of the money supply doesn't effect real variables.

How could inflation impact real variables?

- The key is the timing of events in the monetary model.
- The decision to consume today or tomorrow us determine by

  $MU(c_1)/ MU(c_2)=MRS_{c1,c2}=(1+r)$

- The work – leisure decision period is changed because you get paid at the end of the period and can't spend the money until next period.
- That means you give up leisure today which costs you MU_{l1}..

- The benefit from working is the nominal wage, Pw, and the consumption it buys you tomorrow
- The marginal condition:

$MU(l_1)=MU(c_2)*[P_1w/P_2]$

Or,

$MU(l_1)/MU(c_2)=MRS_{l,c1} = [P_1w/P_2]$

- We need to do some more algebra.
- $MRS_{l,c1} =[MU(l_1)/MU(c_2)][MU(c_1)/MU(c_1)]$

$=[MU(l_1)/MU(c_1)][MU(c_1)/MU(c_2)]$

$= [P_1w/P_2]*(1+r)$

$= w/[(P_2/P_1)*(1+r)]$

$= w/(1+R)$

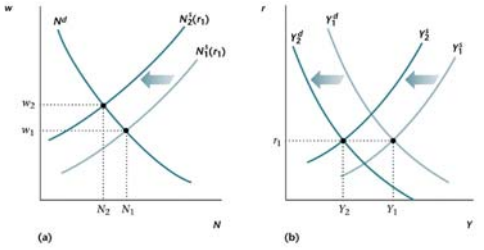
- Substitute for w

$MRS_{l,c1} = w/(1+R)$

$MRS_{l,c1} = MRT_{l,c1} /(1+R)$

- One of the key equations for the pareto optimality of the competitive equilibrium no longer holds unless R equals zero!
- If R>0, then too much leisure is being taken and too little output is being produced. Inflation has costs.

Figure 15.5 The Long-Run Effects of an Increase in the Money Growth Rate



- What does this all mean for monetary policy?
- The optimal monetary policy is to set R=0. This means if

  R = r+I = r+x

  x = -r for R to be zero

- It is optimal for the government to generate a deflation that continues forever.
- This is the Friedman Rule.

Why isn't a Friedman Rule followed in practice?

- The welfare costs in small inflations are small.
- Monetary policymakers worry about liquidity traps? How do you stimulate the economy if the nominal interest rate is zero?