

Bidding in Sealed Bid Auctions

Where the Number of Bidders is Unknown

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Abstract

We analyze experimental data from a series of first price (FP) and second price (SP) sealed-bid auctions in which the number of bidders is unknown. In SP auctions we find a substantial amount of coincidence with theory. We observe systematic deviations from risk neutral bidding in FP auctions and show that these deviations are consistent with risk averse preferences. Employing non-structural latent class estimation methods we find essentially no heterogeneity in bidding in SP auctions, while in the FP auctions heterogeneity in bidding persists and increases with experience. This result is consistent with heterogeneous risk averse preferences and we therefore compare non-structural classification with independently estimated distribution of individual risk parameters. We conclude that heterogeneity in bidding in FP auctions is consistent with heterogeneity in risk preferences, the attempt to count the number of bidders in the auction, and bidder specific noise. (*JEL D44, C91*)

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1. INTRODUCTION

Most of the work in sealed bid auction theory and related experimental work has focused on the case in which the number of bidders is known. This is, of course, not always the case in naturally occurring environments. For instance, construction firms submitting bids in a sealed-bid procurement auction may not know exactly how many other firms have completed the costly process of preparing and submitting a bid until bidding has closed. There is a body of theory to address environments with an unknown number of bidders (McAfee and McMillan (1987), Matthews (1987), Harstad, et al. (1990) and Krishna (2002)).¹ The purpose of this paper is to provide theoretical considerations and analysis of a large dataset of individual bids from laboratory sealed bid auction markets in which buyers have some information about the number of rivals, but not certainty.

According to the received theory, the availability of information about the number of rivals, as expected, directly affects bidding and therefore seller revenue in first price auctions but should not affect bidding in the second price auctions. Although bidding own valuation is optimal in second price auctions (regardless of the number of bidders), the majority of previous experimental work on auctions with known number of bidders has shown systematic deviation (see Kagel, 1995, etc.) while a few studies report coincidence with theory (see Isaac and James (2000b) for an extended discussion). Earlier work on first price auctions with known number of bidders indicated systematic overbidding compared to risk neutral equilibrium. Risk aversion has been suggested as one of the possible explanations for observed overbidding (see Cox et al, 1982, etc.).

Dyer et al. (1989) is the only previous experimental study looking at individual bidding in auctions where the number of bidders is unknown and modeled in a Bayesian way (i.e. as risk and not as ambiguity).² Their setup is a special case of our design where the number of bidders is obtained from a known distribution. We show that in this environment bidding above risk neutral equilibrium is consistent with risk averse preferences. While we analyze the bidding strategy in a more general setting, our study also addresses a broad set of questions.

Some of these questions are motivated by previous studies of open-bid ascending “silent” auctions that suggested revenue loss to the seller by bidders “guarding” their higher valuing items, i.e. remaining physically proximate to the bidding stations (Isaac and Schnier, 2005). The possibility that “guarding” was

¹ A recent paper by Levin and Ozdenoren looks at how bidders and seller respond to ambiguity about the number of bidders.

² They look at FP auctions with restricted auction size of either 6 or 3 participants and ask subjects to submit 3 bids (2 bids for contingent bidding procedure and one bid for non-contingent bidding procedure). Subjects submit all 3 bids simultaneously and do not know at the time which procedure will be used. They find overbidding in this environment compared to risk neutral Nash equilibrium. In our study the number of active bidders in each auction is drawn via known stochastic process with support $[0, 8]$.

at the root of the revenue reductions was investigated by Isaac and Schnier (2006), who replaced the open-bid ascending silent auctions with FP and SP sealed bid auctions. Isaac and Schnier (2006) reported on the aggregate revenue effects of the switch to sealed bid auctions, which were consistent with the notion that sealed bidding eliminated some of the revenue losses. However they did not analyze the causes of such revenue change which are determined by individual bidding behavior. Any analysis of individual bidding behavior in this environment must be informed by theories of bidding with an unknown number of rivals, because such is the reality facing bidders in a sealed bid silent auction context. We present such an analysis of that data in this paper.³

Our analysis of individual bidding data contributes to the discussion about the causes of heterogeneous deviation of individual bidding from risk neutral theory in FP auctions. We employ non-structural latent class estimation methods which do not require any assumptions on the nature or causes of heterogeneity and find essentially no heterogeneity in bidding in SP auctions. In the FP auctions heterogeneity in bidding persists and increases with experience. Cox et al. (1982, 1988) reported heterogeneous bidding above risk neutral equilibrium in FP auctions for mechanisms with fixed number of bidders and Palfrey and Pevnitskaya (2008) for auctions with endogenous entry, where the number of bidders was known at time of submitting a bid. Such deviations would be consistent with heterogeneous risk averse preferences. We therefore independently estimate distribution of individual risk parameters using structural setup in FP auctions and compare the results to non-structural classification described above. We conclude that heterogeneity in bidding in FP auctions is consistent with heterogeneity in risk preferences, the attempt to count the number of bidders in the auction, and bidder specific noise.

In Section 2, we present theoretical results on bidding in silent auctions. Section 3 offers experimental design and hypotheses. Estimation methods are described in Section 4. The experimental results appear in Section 5. Section 6 offers our conclusions.

2. A THEORY OF BIDDING WHERE “ n ” IS UNKNOWN

In the sealed-bid silent auctions bidders do not know at the time they are bidding on any specific item how many of the other bidders will be active rivals. Specifically at the time of submitting a bid, each bidder knows the auction mechanism (here first price or second price), their own valuation of the item, the distribution of valuations of other bidders and the distribution of the number of bidders in the auction (here

³ One of our main objectives is therefore to explore the usefulness of the received theory in an environment that is a closest approximation of the field; our research is not specifically designed to test the theory in the sense of giving it its "best shot."

over $[L, N]$). We start with presenting risk neutral theory of bidding in this environment drawing upon McAfee and McMillan (1987), Harstad, et al. (1990) and Krishna's (2002).

Not surprisingly, having an unknown number of active bidders, n , out of the N total bidders does not affect the optimal bidding strategy, $B(\cdot)$, for second-price sealed bid auctions, namely

$$B(v_i) = v_i \quad (1)$$

where v_i is bidder i 's valuation.

For first-price sealed bid auctions, however, the situation is much different. The Nash equilibrium bid function for risk-neutral bidders is:

$$\beta_{RN}(v_i) = \sum_n \omega_n(v_i) \beta_n(v_i) \quad (2)$$

where $\beta_n(v_i)$ is a risk neutral bid in an auction with n bidders and the weights, $\omega_n(v_i)$, are functions of the bidders expectations $\{\rho_k\}$, which are the probabilities that the bidder is in an auction with k bidders, $\sum_{k=1}^N \rho_k = 1$.⁴ That is, the equilibrium bid function when the bidder "is unsure about the number of rivals he faces is a weighted average of the equilibrium bids in auctions when the number of bidders is known by all" (Krishna, p.36).

Overbidding in first price single unit auctions with known number of bidders has been observed in multiple studies with risk aversion being one, but not the only, explanation (see Cox, Roberson and Smith, (1982) and Kagel (1995) for the discussion). However the recent models of bidding in first price auctions where the number of bidders is uncertain focused only on risk neutral participants.⁵ In this section we derive properties of the bidding functions when bidders are risk averse.

Suppose that subjects have income utility $U(x)$, a concave, increasing, differentiable function satisfying $U(0)=0$. The symmetric Nash equilibrium bid maximizes

$$\max_{\beta} \sum_{j=1}^m p_j U(v_i - \beta(v_i)) F^{j-1}(v_i).$$

where p_j is the probability of participating in an auction with j bidders and $F(v)$ is the cumulative distribution function of bidders' valuations. The optimal bid satisfies the first order conditions for maximization resulting in the following differential equation:

⁴ Krishna's notation is stated in terms of "number of other bidders." Note that in our experiments, these probabilities reflect the stochastic process from determining both how many bidders there will be in a particular auction and then the assignment of bidders with positive values to those auctions. Bidders "not in the auction" are actually assigned a value of zero.

⁵ Matthews compares two auction mechanisms for buyers with CARA, DARA and IARA preferences.

$$\beta'(v_i) = \sum_{j=1}^m \frac{U(v_i - \beta(v_i))}{U'(v_i - \beta(v_i))} (j-1) \frac{F^{j-1}(v_i) p_j}{\sum_j p_j F^{j-1}(v_i)} \frac{f(v_i)}{F(v_i)} = \sum_{j=1}^m \frac{U(v_i - \beta(v_i))}{U'(v_i - \beta(v_i))} (j-1) w_j(v) \frac{f(v_i)}{F(v_i)}$$

where the weights $w_j(v) = \frac{F^{j-1}(v) p_j}{\sum_j p_j F^{j-1}(v)}$. The equilibrium bid function however is no longer a weighted

combination of bids in corresponding j -bidder auctions. Krishna's proof breaks since it relies on revenue equivalence which no longer holds. The Harstad, et al. derivation relies on linear utility and also no longer holds. Despite this we can compare bidding functions of risk averse, $\beta(v)$, and risk neutral, $\beta_{RN}(v)$, bidders.

Proposition: $\beta(v) > \beta_{RN}(v)$

Proof:

Following Harstad, et al. the first order conditions of the maximization problem of risk neutral bidders results in the following differential equation:

$$\beta_{RN}'(v_i) = \sum_{j=1}^m (v_i - \beta_{RN}(v_i))(j-1) \frac{F^{j-1}(v_i) p_j}{\sum_j p_j F^{j-1}(v_i)} \frac{f(v_i)}{F(v_i)} = \sum_{j=1}^m (v_i - \beta_{RN}(v_i))(j-1) w_j(v_i) \frac{f(v_i)}{F(v_i)}$$

Therefore inequality $\beta'(v_i) \geq \beta_{RN}'(v_i)$ follows from the concavity of $U(x)$. Following Milgrom and Weber (1982) we conclude that whenever $\beta(v_i) \leq \beta_{RN}(v_i)$, $\beta'(v_i) > \beta_{RN}'(v_i)$; the equilibrium boundary condition is $\beta_{RN}(\underline{v}) = \beta(\underline{v}) = \underline{v}$. It then follows (Milgrom and Weber, Lemma 2) that for $v > \underline{v}$, $\beta(v) > \beta_{RN}(v)$. \square

That is, risk-averse preferences result in overbidding compared to risk neutral behavior in first price auctions when the number of bidders is unknown.

3. EXPERIMENTAL DESIGN AND HYPOTHESES

We report the results from a total of six experimental sessions, each consisting of five auction periods. The two bidding institutions were alternated to allow both within-subject and across-subject comparisons. The five bidding periods of three experimental sessions were sequenced in an FFSSF format; the other three sessions were sequenced SSFFS. Table 1 is a "road map" to the features of this experimental design.

In each period, there were sixteen items offered for sale and eight potential bidders. For each item in each period, the number of active (non-zero value) bidders, between 2 and 8 inclusive, was randomly chosen with a uniform distribution. Then, the subject IDs to receive the positive values were randomly

chosen. Finally, each bidder with a positive value for a given auction had their value randomly chosen over the interval (0.00, 20.00] using a uniform distribution. The subjects were informed of these random processes.

Each item was auctioned at a separate station and a physical separation of the bidding stations for different items, just as one would find in naturally occurring silent auctions, was included in the laboratory design. At each bidding station was placed a “supersized” foam coffee mug with an opaque, slotted top. The letter of the associated item was clearly marked on each mug. Each bidder had a pad of bidding slips on which they were required to write their bidder ID, the item being bid on, their bid, and the time (of seven minutes) remaining on the clock.⁶ They inserted the bidding slip into the mug when they wished to bid for an item.⁷ The Instructions of the experiment can be found in the Appendix.⁸

For the second price auctions, the formal baseline hypothesis is simple: bidders will follow their dominant strategy and bid their value. The experimental auction literature provides a well-known counter-hypothesis, that bidders will generally over-bid their values in a second price sealed bid auction (Cooper, 2008, provide a recent continuation of that literature).

Figure 1(a,b) displays graphs of the first price auction risk neutral equilibrium bidding function, $\beta_{RN}(v_i)$, for the parameters of this experimental design ($r=1$).⁹ It should be noted that the bid function here, unlike the standard case of known n , is non-linear in v_i . As shown theoretically in the previous section, risk averse bidders ($0 < r < 1$) are expected to bid above $\beta_{RN}(v_i)$.

In the case of simultaneous sealed bid silent auctions, there is a plausible alternative to this theory, which we will call the *counting conjecture*. In this naturally occurring institution, the sealed bid silent auction, it is at least possible that bidders will try to watch other bidders and discern the number of bidders in each of the individual auctions. If so, then the silent auctions are not a part of the standard theoretical framework. These auctions would simply devolve to a series of simultaneous first price auctions, with bidders bidding according to the formula based on known n . Of course counting is difficult: bidders must attend to their own decisions and move between different stations; by construction some bids will be placed after others, requiring extensive updating by counters; watching all of the possible rivals may be difficult as

⁶ Only one winning bidding slip (out of 480) was invalidated due to absent information. In addition, there were six non-winning bidding slips invalidated for various reasons of absent or incorrect information whose status cannot be determined (out of a total of 2205).

⁷ In order to keep bidders from wanting to pry open the slots in order to see if they were the only bidder, each mug always contained at least one yellow bid slip marked “blank.” Subjects knew this, and in fact a volunteer monitor from the population of subjects (undergraduate students) saw us put one in each mug.

⁸ IS06 provide more details of the conduct of the experiments, such as the details of the geographic dispersion of the items, and so-forth.

⁹ Theoretical bid functions were obtained using CRRA utility function of the form $U=x^r$, so $r=1$ is risk neutrality.

they span out across the room and each bidder may place a new bid before the auction ends.¹⁰ Therefore, we conjecture that counting, if it occurs, is likely to occur in a context that is between the pure conditions of the theories of known and unknown bidders.

The previous discussion is intended to emphasize that we came upon this context for the applicability of the relevant theory from the direction of the re-design of actual institutions. The non-computerized, geographically dispersed silent auctions would not be the design that we would have chosen if we were following a more traditional route of giving the theory “its best shot.” Nevertheless, the context of the sealed bid silent auctions is close enough to the conditions of the theory to conjecture that it can be a useful tool for organizing the data. At the same time, there are enough unique features of the silent auction to make this an interesting, potentially falsifiable, test.

4 . ESTIMATION METHODS

4.1. General analysis of bidding

For each auction institution we have a relatively robust number of observations. Because the valuations for the items are independently drawn private values, we treat auctions independently. To investigate whether or not bidder behavior is in accordance with the theoretical predictions we estimate a reduced form bid function under both homogeneity and heterogeneity assumptions. The homogeneity assumption implies that all bidders possess the same marginal bidding propensities (regression coefficients), whereas the heterogeneous model allows for there to exist multiple bidding functions within the population. This was achieved utilizing latent class regressions, which have proven to be useful in public good (Anderson and Putterman, 2006) and resource extraction experiments (Schnier and Anderson, 2006). This method will be further outlined following a general discussion of the bid function estimated.

The bid function we estimate for each auction institution is

$$b_{ikt} = \gamma_0 + \gamma_1 * Predicted_{ikt} + \gamma_2 * Number_{kt} + \gamma_3 * New + \gamma_4 * New * Predicted_{ikt} + \gamma_5 * New * Number_{kt} \quad (3)$$

where b_{ikt} represents participant i 's bid on an item with value k in time period t , within the panel for either the first-price or the second-price auction. Within the second-price auction the variable $Predicted_{ikt}$ is bidder i 's value for item k in period t , $Value_{ikt}$, which in theory is equal to the bid. In the first-price auction we use bidder i 's predicted risk neutral equilibrium bid for item k in period t . The variable, $Number_{kt}$ indicates the number of active bidders for item k in period t and is included to investigate the counting

¹⁰ A second alternative is that bidders will use a rule of thumb rather than the non-linear bid function implied by the theory. The most obvious candidate is that they will work with the median numbers of bidders, $n=5$, and hence will bid 80 percent of their value $((n-1)/n)$. We ran a parallel series of estimations to those reported here using this behavioral benchmark. A later footnote explains their similarity to the main results.

conjecture discussed earlier. *New* is a dummy variable indicating whether or not participant *i*'s bid occurred during their first exposure to either auction institution.¹¹ Should bidder behavior follow the theoretical predictions we would expect $\gamma_0 = 0$, $\gamma_1 = 1$, and $\gamma_2 = 0$. If the counting conjecture is to be supported we would require a non-zero and significant coefficient γ_2 . If individual *i*'s behavior requires some learning with the institution, we expect some of γ_3 , γ_4 , or γ_5 to be non-zero. To estimate this bidding function we utilize a double censored tobit regression with an upper bound of \$20.00, the maximum induced value in the experiment, and a lower bound of \$0.00, the minimum induced value.¹²

For the subset of “experienced” bids equation (3) degenerates to

$$b_{ikt} = \gamma_0 + \gamma_1 * Predicted_{ikt} + \gamma_2 * Number_{kt} \quad (4)$$

since *New*=0 for all “experienced” bids. We estimate (4) to investigate the effect of experience on bidding and heterogeneity by comparing the results to the estimation of equation (3). Regression results for equations (3) where we use all data are denoted *New*=0, 1 results, indicating the presence of the *New* dummy variable, and for experienced bidding when *New*=0, used in equation (4), the results are denoted *Experienced*.

4.2. Presence and Degree of Heterogeneity in Bidding

We next test for the presence and degree of heterogeneity in bidding. To conduct the heterogeneous estimation we use El-Gamal and Grether's (1995, 2000) estimation classification (EC) algorithm to endogenously group subjects into a pre-specified number of bidder types.¹³ The EC algorithm assumes that each subject's bids within the experiment can be described by a bidding function, $b(\gamma)$, such as expressed in equation (3), where γ is an unknown parameter vector. This is the same as a traditional regression model, but heterogeneity is introduced by allowing for there to exist a pre-specified number of different “types”, H , with each “type” possessing their own parameter vector γ_h . The estimation of all $h=1, \dots, H$ types' parameter vectors is conducted simultaneously with the determination of each subject's type. This is achieved by having each subject's contribution to the likelihood function, given $\Gamma=(\gamma_1, \dots, \gamma_h)$, be the maximum of the joint likelihood of all their bid observations, n , across the H types.¹⁴ The log-likelihood

¹¹ *New*=1 for the first round decisions with the particular auction mechanism.

¹² The likelihood function for the tobit estimation was programmed in GAUSS and we utilized the MAXLIK toolbox to obtain our parameter estimates.

¹³ Bidder heterogeneity could be controlled for using either a fixed-effects or a random-effects tobit regression. However, this estimation technique would not allow us to determine whether or not individual subjects (or groups of subjects) possessed alternative bidding functions which are statistically different from others within the experiment.

¹⁴ This method may generate a likelihood function which possesses a number of local maxima. To ensure that our parameter estimates are the global maxima we estimate the first-price model 2000 times and the second price model 500 times using different random starting parameters.

function utilizing the EC algorithm and the log-likelihood for the tobit regression, denoted by $L(\cdot)$, is expressed as,

$$\ln[L(b; X | \Gamma, H)] = \sum_{j=1}^m \arg \max_h \left[\sum_{t=1}^T \sum_{i=1}^n \ln[L(b_{itj}; X_{itj} | \gamma_h)] \right].$$

where m is the number of subjects in the experiment (48) and X_{itj} is a matrix of independent variables captured in the bidding function (1). The estimates of Γ are subject to the assumptions made regarding how many “types” there exist within the population. To determine the appropriate number of “types” we utilized the Bayesian information criterion (BIC), the corrected Akaike information criterion (crAIC) and likelihood ratio (LR) tests for alternative H classifications. The results of these estimations are presented in tables 2, 4 and 5. In each table we show homogeneous estimates ($H=1$) as a reference point and then report results from the best fitted number of segments. Model test statistics are presented in Table 3.

4.3. Risk Preferences.

We further explore individual risk aversion by estimating risk aversion coefficients of individual subjects and segments and compare our findings to previous studies and results from non-structural techniques described above. For estimation purposes we adopt CRRA utility function $u(x)=x^r$, where r is the risk parameter such that $r=1$ for risk neutral bidders, $0 < r < 1$ for risk averse bidders and $r > 1$ for risk loving bidders. For CRRA utility function and given our experimental design, the differential equation for the optimal bid becomes:

$$\beta'(v_i) = \frac{1}{rv_i} (v_i - \beta(v_i)) \frac{\sum_{j=2}^8 (j-1) \left(\frac{v_i}{2000}\right)^{j-1} p_j}{\sum_{j=2}^8 \left(\frac{v_i}{2000}\right)^{j-1} p_j}.$$

Although the closed form solution of optimal bid cannot be obtained, we are able to solve the above differential equation numerically for each valuation and risk parameter. Therefore we can obtain a theoretical bid for every valuation (given r) and compare it with the observed bid. We then estimate risk parameters in the following way. For each experimental observation of a bid, we obtain a numerical solution of the optimal bid based on r . We then obtain the difference between theoretical and actual bids. Our estimate of r minimizes the sum of absolute deviations between theoretical and observed bids. We are interested in the following estimators. First we obtain the estimator for the whole population of bidders:

$$r = \arg \min \sum_{i=1}^{48} \sum_{j=1}^{n_i} |\beta^*(v_{ij}, r) - \beta_{ij}|,$$

where v_{ij} is the valuation of bidder i in their auction j , n_i is the number of auctions that bidder i participated in, $\beta^*(v_{ij}, r)$ is the optimal theoretical bid (given r and v_{ij}) and β_{ij} is observed bid for value v_{ij} . For the whole population the estimate $r=0.375$ minimizes the above function. This value is similar to those observed in many previous studies (see, for example, Isaac and James (2000)). The estimates of risk parameters for each group/segment, G_k , described above are

$$r_{G_k} = \arg \min \sum_{i \in G_k} \sum_{j=1}^{n_i} |\beta^*(v_{ij}, r) - \beta_{ij}|.$$

Correspondingly the estimates of individual risk aversion parameters are

$$r_i = \arg \min \sum_{j=1}^{n_i} |\beta^*(v_{ij}, r) - \beta_{ij}|, i=1, \dots, 48.$$

The results of these estimations are presented in Tables 7 and 8.

5. RESULTS

We have 1050 and 1013 observations on bidder's value and their corresponding bid in the second-price and first-price auctions respectively.¹⁵ We next describe our specific findings for second-price and first-price auctions.

5.1 Second-Price Auctions.

From a purely descriptive point of view, we note that 547 (52.09%) of the bids are equal to value, and that 740 (70.48%) were within twenty-five cents of value.¹⁶ This compares favorably to other research into dominant strategy mechanisms and is consistent with Isaac and Schnier (2006) who report that average aggregate revenue was very close to the theoretical predictions for these auctions.¹⁷ This result seems robust in our study and is not driven by previous experience in FP auctions

¹⁵ The following data were excluded relative to the initial valid data set. First, we excluded all bids from the second period of the second FFSSF experiment as one bidder clearly acted upon a misunderstanding of the instructions. Also, bidders could make multiple bids, with the highest bid upon closing being the "official" bid. However, bidders took little advantage of this option as there were only 62 multiple bids, almost evenly split between the first price auctions (33) and the second price auctions (29). We removed these 62 bids for this data analysis. We have conducted all the tests reported here including these bids and all of the main conclusions reported here remain unchanged.

¹⁶ We report results in terms of experimental dollars (one experimental dollar was \$0.25). The bidder who had misunderstood the instructions in an early round was subsequently in the first stage of our bankruptcy rule, which restricted bidding over value. This was the only bidder to enter any stage of the bankruptcy rule. Removing all the data for this bidder yields revelation percentages that are actually higher, 53.26% (exact) and 71.86% (within 25 cents) respectively. This bidder persistently bid *below* value in the second-price auctions.

¹⁷ This result is robust in our study and is not driven by previous experience in FP auctions. When we separately analyze bidding in SP auctions prior to exposure to FP auction in the SSFFS treatment (dropping New data) we still find that 82.79% of bids are within 25 cents of valuation.

Turning to the estimation results, we note that the econometric firepower of the latent class process was essentially wasted on the second-price auctions, as both the full data set and the data restricted to experienced periods failed to converge beyond a single class.¹⁸ Table 2 shows that for the $New = 0, 1$ case subjects appear to be following the theoretical predictions except for when they are initially exposed to the second price auction institution. The negative coefficient on the $New*Value$, indicates that subjects bid more than 9% below their value when first exposed to new auction institution. The $Value$ coefficient is not statistically significant from its hypothesized theoretical value (using a t -test).¹⁹ The bidding behavior does not depend on the number of bidders in the auction. This is not surprising because in the second price auctions the optimal bid does not depend on n and the *counting hypothesis* is irrelevant in this case.

Given the statistical significance of some of the “*New*” variables, we re-estimate the results with only the experienced bidding. The heterogeneous estimation again collapses to a single class outcome. As shown in the right hand column (*Experienced*) of Table 2, we find that experienced subjects bid almost exactly like the dominant strategy prediction. A t -test on the $Value$ coefficient, the only statistically significant coefficient in the experienced model, confirmed this hypothesis.²⁰ What chiefly distinguishes the full data set from that restricted to the experienced periods is the much smaller variance around the estimates in the latter model.

5.2 First Price Auctions.

Table 3 reports the test statistics for the heterogeneity in bidding for $New = 0, 1$ and *Experienced* data. The test criteria clearly show that for heterogeneous estimations of the first-price auctions with $New = 0, 1$, a homogenous model is inadequate, with a three segment model performing best. Table 4 presents estimation results for homogenous ($H=1$) and heterogeneous (optimal $H=3$) specifications of the model on all $New = 0, 1$ data. The results from the homogeneous estimation ($H=1$) indicate that subjects, treated as one group, do not behave according to the equilibrium theoretical predictions. Three of the coefficients are statistically different from their hypothesized values. The constant term is positive and statistically significant. Subjects, as an aggregate, are influenced by initial exposure to the auction institution and the significant coefficients on the New and the $New*Predicted$ variables indicate a structural shift in their bid function. The coefficient on the $Predicted$ variable, however, is not significantly different from 1.00 as suggested by theory.

¹⁸ In fact, the tight fit of the data to the theoretical model apparently caused problematic convergence in GAUSS for even the single segment model. The estimates we report in that one case are instead from a STATA estimation.

¹⁹ The t -test yielded a value of 0.702.

²⁰ The t -test yielded a value of 0.692.

The heterogeneous results for the $New=0,1$ model differ from those obtained with a homogenous model. A majority of the subjects (35), those in Segments 1 and 2, tend to bid above the equilibrium theoretical predictions. T-tests on the *Predicted* coefficient indicate that this coefficient is significantly greater than 1.00 in both cases.²¹ For all three segments the estimated constant is positive, but it is statistically significant (with 95% confidence) only for Segment 1. Segments 2 and 3 are distinguished from Segment 1 also by the fact that the former show stronger structural shifts from the *New* condition. In no case is there any significant evidence of the counting conjecture.

The 12 bidders in Segment 1 have a propensity to bid a large predetermined amount, as reflected in the large and statistically significant constant term (1.2590) and also depart from risk neutral theory by having a coefficient on *Predicted* (1.1400) that is significantly greater than 1. The 23 bidders in Segment 2 have a much smaller constant term (0.4552) and they come closest to the coefficient of 1.00 on the *Predicted* variable (1.0800). The 13 bidders in Segment 3 are particularly interesting. They have an intermediate constant of 0.8617 (which is not significant for any standard confidence level) and their coefficient on *Predicted* is only 0.9076. The coefficients for this segment tend to have larger standard errors than the other two groups.²²

Table 5 presents the estimations for the *Experienced* data and model specification (4). The homogenous model ($H=1$) yields nearly identical parameter estimates for the *Constant*, *Predicted*, and *Number* coefficients as those depicted for the homogenous case in Table 4. For the heterogeneous case, the criteria in Table 3 support a four segment estimation. Note that for the first price auction heterogeneity increased with experience. Segments 1 and 4 (with 17 and 12 bidders respectively) have a positive (and significant) constant and slightly greater than 1 coefficient on the predicted bid. For segments 2 and 3 (with 15 and 4 bidders respectively) the coefficient on the constant term is not significantly different from zero. Segment 2's *Predicted* coefficient is very close to 1, while for segment 3 it is below 1 and equal to 0.6359.²³ Furthermore, segment 2 has a *Number* coefficient that is significantly different from zero ($p=0.02$), the only time the counting hypothesis is supported in this study!²⁴

²¹ The *t*-tests yielded values of 10.294 and 5.161 for $h=1$ and $h=2$ respectively.

²² We had originally considered what we termed a "rule of thumb" or "behavioral" alternative model for bidding in the first price auctions, namely that bidders would bid the Nash equilibrium bid as though they were in the median size group ($N = 5$). The risk neutral equilibrium version would be a bid of .8 times value. Overbidding would be transparently consistent with risk aversion just as in the standard literature. As it turns out, the results using this behavioral alternative are qualitatively very similar to those we presented here. The results with the linear approximation do indicate a better fit in that the constant terms are smaller and less significant, and they have slightly larger log likelihood. However, the change does nothing to eliminate the apparent overbidding with respect to the prediction, in fact, with respect to the behavioral benchmark, the estimated overbidding is even worse. Furthermore the classification schemes are very close.

²³ Another interesting property of the latent class EC algorithm is its selection into segments using not just average individual bidding level but also bidder specific noise. Segment 3 is characterized by lower bidding and also large deviation and includes

Unlike the second price auctions where the model converged to 1 segment (homogeneity), in the first price auctions heterogeneity increased with experience resulting in 4 segment specification for experienced data compared to 3 segment specification for all data. Such difference in heterogeneity is consistent with heterogeneity in risk preferences which should play no role in the second price auction (where the solution concept is dominance) and directly affect bidding in first price auctions. We study these considerations in detail in the next section.

5.3 Individual characteristics of heterogeneity

We next determine if there is any regularity in how *specific* subjects are assigned to segments. Following methodology of section 4.3 we study whether the pattern of bidding consistent with risk aversion plays a role in segment assignments.

First, consider the second price auctions, where risk preferences play no role for the bidding strategy and the model collapsed to one segment. Using all of the data ($New=0,1$) the point predictions of the coefficients are similar but noisier than estimates for *Experienced* bidding. This suggests the presence of noise in behavior in the face of a new institution, a propensity that does not qualify as risk aversion.

However, the analysis of the first price auctions indicates that heterogeneity persists and increases (or get more refined) with experience. We explore below whether there is a common factor in the segment assignments for the first price auction that is consistent with risk aversion.

We report the estimated risk parameter (as well as standard error and the number of observations) for each individual bidder in Table 7. Estimation results of risk parameters for the optimal segments formed by the EC algorithm are presented in Table 8. Since the segment numbers are arbitrary, we order the results in Table 8 in increasing risk preference parameter, r . We perform these estimations for all data ($New=0,1$) as well as for only experienced bidders' data (*Experienced*). Bidders risk parameters, r , range from 0.035 (subject 27) to 34.9 (subject 7) for all data ($New=0,1$) and from 0.045 (subject 27) to 27.30 (subject 7) for experienced data. Subject 7's bidding is an outlier and is also characterized by the largest noise. The second highest risk preference parameter is 2.55 for all data and 2.38 for experienced data (subject 3). The ranking of individual risk parameters is preserved somewhat well with experience as shown in Table 7. The distributions of the individual risk parameters are displayed in Figure 2. These distributions are obtained by counting the frequency of individual r_i s in the intervals with length 0.1, i.e. from 0 to .1, from .1+ to .2, etc. The arrows line up to the estimated risk aversion parameters of segments from Table 8. The distribution of

(without any ex ante assumptions) subject 7 which is an outlier in terms of both. This feature of the algorithm may be useful in future experimental settings where we do not know the source of heterogeneity or when controlling for outliers is desired.

²⁴ The next smallest p value is 0.31 in segment 1.

individual risk parameters for all data is somewhat uniform with no distinct peaks. The estimated risk parameters of three optimal segments formed by EC algorithm (0.21, 0.52 and 0.99) are spread about evenly among risk averse to risk neutral range of risk parameters. The distribution of individual risk parameters changes dramatically when we look at only experienced bidders and has two very distinct peaks. Segments 1 and 4 (containing a total of 29 of the 48 bidders) line up at the first peak in the distribution and segment 2 (containing 15 out of 48 bidders) lines up near the second peak. Segment 3 is at the right tail and has the highest $r=1.505$ (least risk averse). Bidding in segments 1, 2 and 4 is consistent with risk averse preferences ($r=0.375, 0.505$ and 0.205 respectively). Segment 2 however is the only group to show significant counting behavior. Therefore although a large number of bidders exhibited risk averse risk preferences (if we were to use this hypothesis) the behavioral phenomenon of counting also separates the segments.

Figure 1(a, b) shows bidding data by segments for *Experienced* data and also plots two theoretical benchmarks: risk neutral ($r=1$) bid and bid function based on the optimal r estimate for each segment. The graphs show that different segments have different levels of bidding noise. Data in segments 1 and 4 clusters tightly around theoretical predictions, while data in segment 2 has larger noise. Segment 3 is associated with particularly large bidding noise. Some subject, for example 3 and 7, bid all over the non-dominated range of bids ($b_{ij} \leq v_{ij}$). It thus appears that the estimation technique is sorting the bidders in segments based also on the intrinsic noise.

Table 6 presents the contingency table for the segment assignments in all data and only in experienced periods. The chi-squared statistic for the associated contingency test is significant, indicating there is some common element to the heterogeneity of the bidder behavior that survives when we control for the effect of experience. If risk preferences are stable, there would be higher numbers along the main diagonal and lower numbers or zeros in the lower left and upper right corners. This tendency is observed in the table with a slight deviation, which is due to other factors affecting heterogeneity besides risk preferences. The majority of subjects (23) from the “middle” risk aversion segment 2 ($r=.515$) based on *New=0,1* data remained in “middle” segments 1 and 2 ($r=.375$ and $.505$ respectively) for *Experienced* data. 10 out of 12 subjects in the most risk averse (lowest r) segment of all data ($r=0.205$) remain in the most risk averse segment of experienced data ($r=0.205$). There are only 2 subjects, 9 and 48, who were in Segment 3 ($r=0.985$) for all data and joined Segment 1 ($r=0.205$) for *Experienced* data. As follows from Table 7, the estimated risk parameters for *New=0,1* and *Experienced* data were, respectively, 0.36 and 0.29 for subject 9 and 0.26 and 0.24 for subject 48. Although the estimated risk parameters of these subjects did not change much with experience, the bidding noise (standard deviation from theoretical bid) dropped

significantly: from 2.84 to 0.28 for subject 9 and from 2.00 to 0.26 for subject 48. Table 8 indicates that Segment 3 for all data is characterized by the largest bidding noise (standard deviation from theoretical bid is 4.16). This is yet another indication that bidder specific noise is utilized by the EC algorithm to identify heterogeneity among subjects.

Our results demonstrate that heterogeneity in bidding captured by the latent-class estimation technique is consistent with three characteristics: risk preferences, counting and intrinsic bidding noise of a given subject.

6. CONCLUSIONS

Previous studies of individual bidding data in sealed bid auctions with known number of bidders indicated that bidders deviate from theoretical predictions and exhibit heterogeneity (Cox et al, 1982, Kagel, 1995, etc.). In this paper we study individual bidding behavior in sealed bid auctions with unknown number of bidders (modeling uncertainty in a Bayesian way) and look for the consistency of individual bidding decisions with models of individual bidding behavior. Previous theoretical work in this environment identified the dominant strategy in the second price auctions and risk neutral equilibrium model in the first price auctions, which we adapted to our experimental design. We then extended existing risk neutral theory of bidding in first price auctions with unknown number of bidders to account for risk preferences. The permutations on the basic models that we considered were first, whether, there was any indication that bidders could discern (count) the number of bidders in each of the auctions; second, an experience effect; and third whether there might be individual heterogeneity and the possibility that it could be explained by heterogeneous risk preferences.

We found solid support for the proposition that bidders come close to the dominant strategy of truthful revelation in second price sealed bid auctions, especially with a modest amount of repetition. To the extent that these results differ from other studies, further experimental examination of the causes should provide fertile ground for exploration.

In the first price auctions the majority of bidders demonstrate a modest amount of overbidding compared to risk neutral model. We show that this over-bidding is consistent with risk aversion. The latent class estimation process yields segment assignments that are in general consistent with the patterns of estimated individual and segment risk aversion, especially for experienced bidders. However, there is evidence for bidder heterogeneity driven by components beyond the standard model of risk aversion. The number of segments increased with experience in the first price auctions and the assignments in the *New = 0,1* case and the *Experienced* case are not identical, indicating heterogeneity that changes with experience

in the institution. Bidding noise differs across segments and tends to decrease with experience in both first price and second price auctions. We find that the model converged to relatively homogeneous behavior in the second price auctions for both the full and experienced data sets, *New=0,1* and *Experienced* respectively. However in the first price auctions heterogeneity persisted (and slightly increased), while the bidding noise decreased with experience. One of the characteristics affecting bidding in the first price auctions is the phenomena of counting the number of bidders, which is significant in only one segment. Segment assignments are also influenced by the magnitude of bidder specific noise (which is heterogeneous).

These results provide an interesting menu for further research on auctions with an unknown number of bidders. Two obvious areas are: i) a switch to common and affiliated as opposed to independent private values; and ii) allowing bidders to choose bidding in first price, second price, or ascending auctions (see, for example, Ivanova-Stenzel and Salmon 2005) and analyzing comparative performance. The results presented here also inform the longstanding debate on the role of risk aversion in first price auction overbidding and bidding heterogeneity. A study determining the extent to which the factors that we identified explain heterogeneity would provide a further contribution to the theory of individual behavior in auctions.

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Table 1. Experimental Design.

Institution	No. of Sessions	Period 1	Period 2	Period 3	Period 4	Period 5
FFSSF	3	First Price	First Price	Second Price	Second Price	First Price
SSFFS	3	Second Price	Second Price	First Price	First Price	Second Price

Table 2: Latent Class Regressions – 2nd Price Auction.^a

Variable	<i>New = (0,1)</i> <i>H=1</i>	<i>Experienced</i> <i>H=1</i>
<i>Constant</i>	0.0292 (0.14)	-0.0791 (-0.981)
<i>Value</i>	1.0071** (99.01)	1.0027** (259.677)
<i>Number</i>	-0.0188 (-0.61)	0.0029 (0.242)
<i>New</i>	-0.1746 (-0.52)	
<i>New*Value</i>	-0.0939** (-5.91)	
<i>New* Number</i>	0.0818 (1.61)	
σ^2	1.4789	0.5617
# in segment	48	48
Log-Like.	-1887.89	-604.33

a: t-statistics for the regression coefficients are shown in parenthesis

** - statistically significant at 95% confidence, t-test

* - statistically significant at 90% confidence, t-test

Table 3: Model Test Statistics

Institution/Data Set	<i>New=0,1</i>			<i>Experienced</i>		
	LR	BIC	crAIC	LR	BIC	crAIC
First Price						
<i>H=1</i>	-----	4789.89	4825.87	-----	2450.62	2460.47
<i>H=2</i>	877.66	3912.23	4146.36	561.10	1889.52	1939.54
<i>H=3</i>	316.46	3595.77	4483.98	155.51	1734.01	1882.61
<i>H=4</i>	174.01	3421.73	6180.53	58.71	1675.30	2029.83

Table 4: Latent Class Regressions – 1st Price Auction: 3 Segment Model ($New=0,1$).^a

Variable	Homogeneous	Heterogenous		
	$H=1$	$H=3; h=1$	$H=3; h=2$	$H=3; h=3$
<i>Constant</i>	0.9389** (2.52)	1.2590** (6.13)	0.4552* (1.733)	0.8617 (1.733)
<i>Predicted</i>	1.0351** (46.242)	1.1400** (83.73)	1.0800** (69.66)	0.9076** (18.43)
<i>Number</i>	0.0125 (-0.22)	-0.0066 (-0.22)	0.0605 (1.55)	0.0524 (0.37)
<i>New</i>	1.1010** (2.00)	-0.0518 (-0.17)	0.3452 (2.19)	1.4123 (0.91)
<i>New*</i> <i>Predicted</i>	-0.1834** (-5.34)	0.0175 (0.92)	-0.0819** (-3.45)	-0.5219** (-6.14)
<i>New*</i> <i>Number</i>	-0.1153 (-1.37)	0.0005 (0.01)	-0.0685 (-1.19)	-0.0011 (-0.00)
σ^2	2.5762	0.6613	1.2342	3.3138
# in segment	48	12	23	13
Log-Like.	-2393.01		-1795.94	

a: t-statistics for the regression coefficients are shown in parenthesis

** - statistically significant at 95% confidence, t-test

* - statistically significant at 90% confidence, t-test

Table 5: Latent Class Regressions – 1st Price Auction: 4 Segment Model (*Experienced*).^a

Variable	Homogeneous		Heterogeneous		
	$H=1$	$H=4; h=1$	$H=4; h=2$	$H=4; h=3$	$H=4; h=4$
<i>Constant</i>	0.9438** (3.15)	0.9664** (5.06)	0.2213 (0.68)	0.6209 (1.33)	1.2066** (7.16)
<i>Predicted</i>	1.0349** (56.987)	1.1102** (100.94)	1.0501** (53.08)	0.6359** (8.34)	1.1547** (99.67)
<i>Number</i>	0.0120 (0.270)	-0.0292 (-1.03)	0.1069** (2.25)	0.1252 (0.62)	-0.0043 (-0.17)
σ^2	2.0899	0.7113	1.3221	3.4998	0.5690
# in segment	48	17	15	4	12
Log-Like.	-1223.38		-827.38		

a: t-statistics for the regression coefficients are shown in parenthesis

** - statistically significant at 95% confidence, t-test

* - statistically significant at 90% confidence, t-test

Table 6: Contingency Table of Assignments

(Due to arbitrary assignment of segment numbers we order the segments in increasing r)

First Price
Auction
(*Experienced*)

		Segment 4 ($r=.205$)	Segment 1 ($r=.375$)	Segment 2 ($r=.505$)	Segment 3 ($r=1.505$)	Total
First Price Auction ($New = 0,1$)	Segment 1 ($r=.205$)	10	2	0	0	12
	Segment 2 ($r=.515$)	0	12	11	0	23
	Segment 3 ($r=.985$)	2	3	4	4	13
Total		12	17	15	4	48

Table 7. Estimates of individual risk preference parameters.

Bid ID	<i>Experienced</i>			<i>New=0 and 1</i>		
	<i>r</i>	<i>s dev</i>	<i>N</i>	<i>r</i>	<i>s dev</i>	<i>N</i>
1	1.125	2.223	20	1.515	2.854	27
2	0.805	0.622	14	0.965	0.885	23
3	2.375	1.957	19	2.545	1.765	27
4	0.425	0.973	19	0.425	4.546	30
5	0.205	0.718	20	0.195	0.722	28
6	0.515	1.173	15	0.565	2.001	24
7	27.3	4.370	18	34.90	3.731	25
8	0.295	2.372	19	0.865	2.242	27
9	0.285	0.284	11	0.355	2.836	17
10	0.295	0.524	7	1.395	2.582	16
11	0.185	0.567	9	1.025	1.428	18
12	0.615	0.419	10	0.705	2.589	18
13	0.385	1.118	10	0.455	0.962	23
14	0.425	1.191	9	1.345	3.245	20
15	0.285	0.370	9	0.535	0.769	19
16	0.385	0.683	10	0.675	1.581	17
17	0.505	0.432	21	0.515	1.854	27
18	0.595	0.923	13	0.575	0.898	21
19	0.265	0.581	18	0.335	5.681	26
20	0.235	1.310	19	0.315	2.151	30
21	0.505	0.731	16	0.515	0.987	23
22	0.785	0.748	17	0.785	0.744	27
23	0.895	2.413	17	0.995	2.441	27
24	0.255	1.577	21	0.415	1.409	32
25	0.135	0.213	7	0.135	0.233	16
26	0.345	0.601	10	0.335	0.607	20
27	0.045	0.152	7	0.035	0.144	19
28	0.455	0.741	6	0.455	0.753	7
29	0.515	0.573	7	0.515	0.548	15
30	0.565	1.129	9	0.725	0.910	23
31	0.355	0.418	9	0.265	0.988	20
32	0.305	0.461	11	0.275	0.403	19
33	0.515	1.273	7	0.465	0.895	16
34	0.265	1.885	11	0.265	1.373	21
35	0.115	0.246	7	0.105	0.193	19
36	0.625	2.751	9	0.865	5.797	19
37	0.105	1.074	7	0.125	0.727	15
38	0.235	0.134	9	0.205	0.267	22
39	0.265	0.792	9	0.315	1.389	20
40	0.505	1.552	10	0.505	1.306	17
41	0.145	0.406	7	0.185	0.410	15
42	0.255	0.404	11	0.215	0.362	21
43	0.325	0.861	6	0.515	0.864	18
44	0.295	0.821	12	0.315	0.681	24
45	0.525	0.601	7	0.515	0.459	13
46	0.235	0.274	9	0.235	0.341	23
47	0.985	1.188	9	0.695	1.387	19
48	0.235	0.264	11	0.255	2.004	20
All	0.375	2.205	568	0.455	2.846	1013

Table 8. Risk preference parameters by segments.
 (segments are ordered in increasing r for presentation; “s_dev” - standard deviation; N – number of observations)

Bid ID	<i>Experienced</i>			<i>New=0 and 1</i>			
	<i>r</i>	<i>s_dev</i>	<i>N</i>		<i>r</i>	<i>s_dev</i>	<i>N</i>
All	0.375	2.205	568	All	0.455	2.846	1013
Segment 4	0.205	0.738	117	Segment 1	0.205	0.652	238
Segment 1	0.375	0.636	177	Segment 2	0.515	1.316	476
Segment 2	0.505	1.423	200	Segment 3	0.985	4.162	299
Segment 3	1.505	3.824	74				

Figure 1(a). First Price Auction Bidding

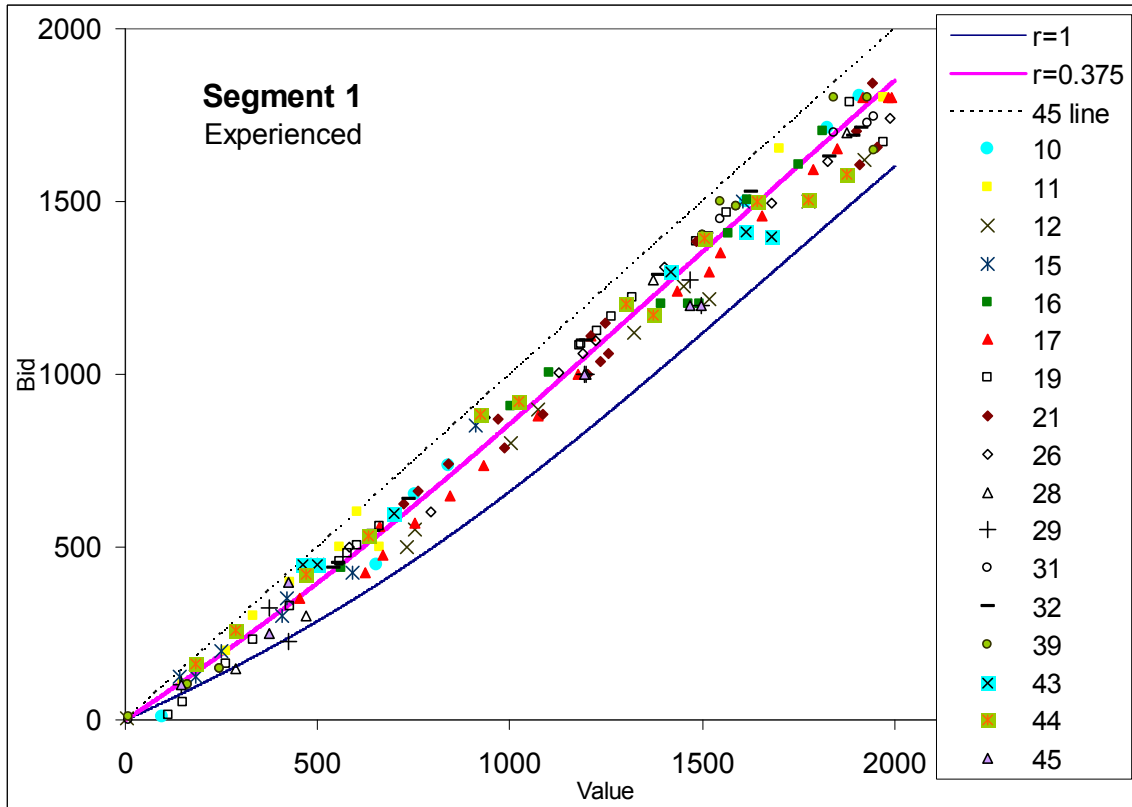
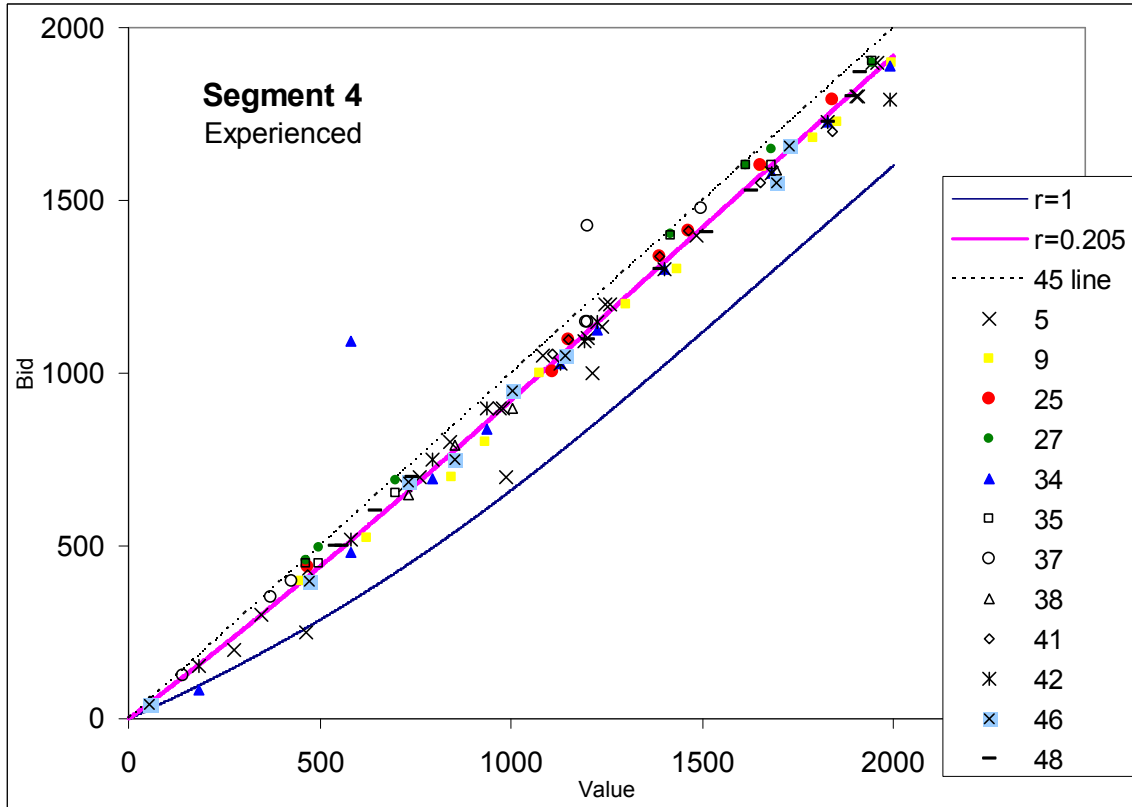


Figure 1(b). First Price Auction Bidding

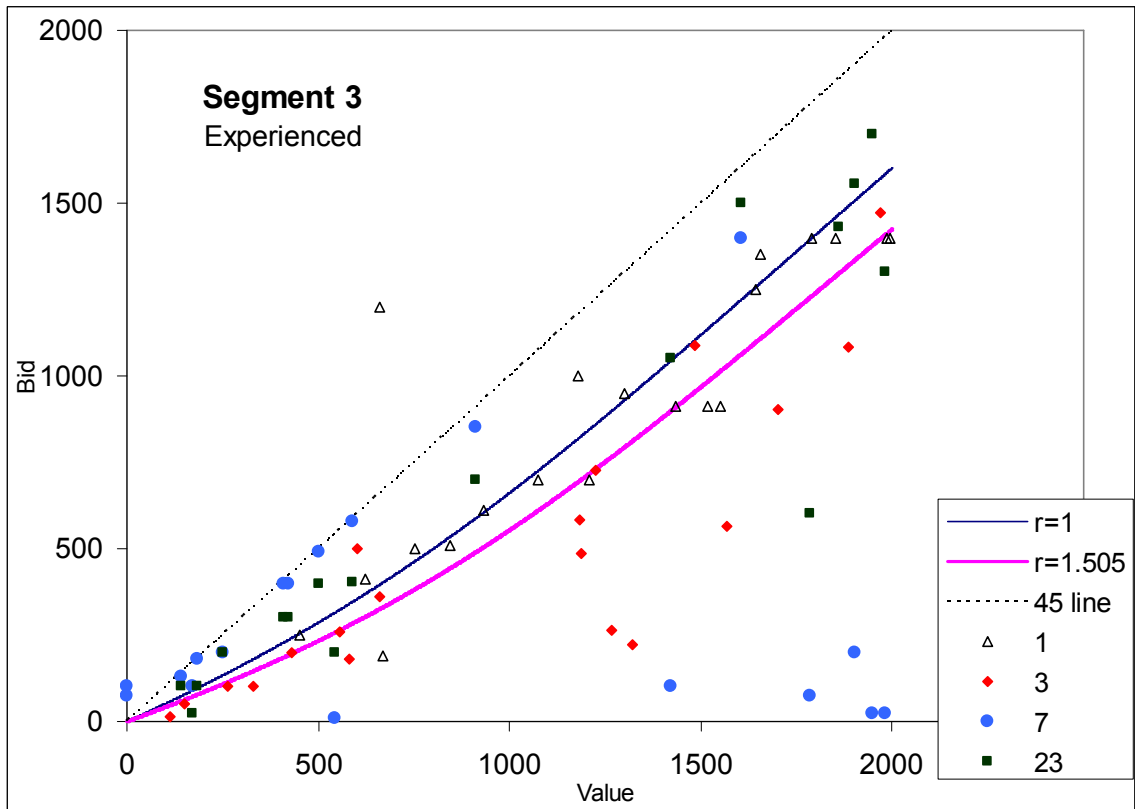
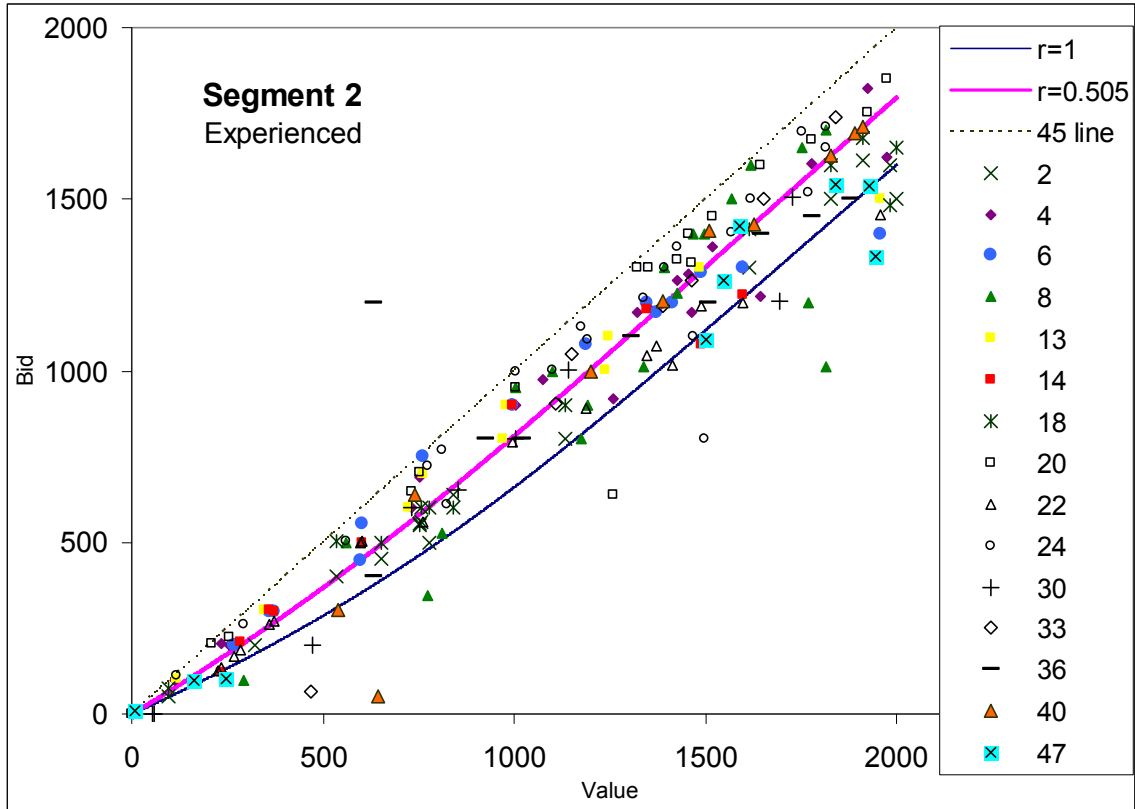
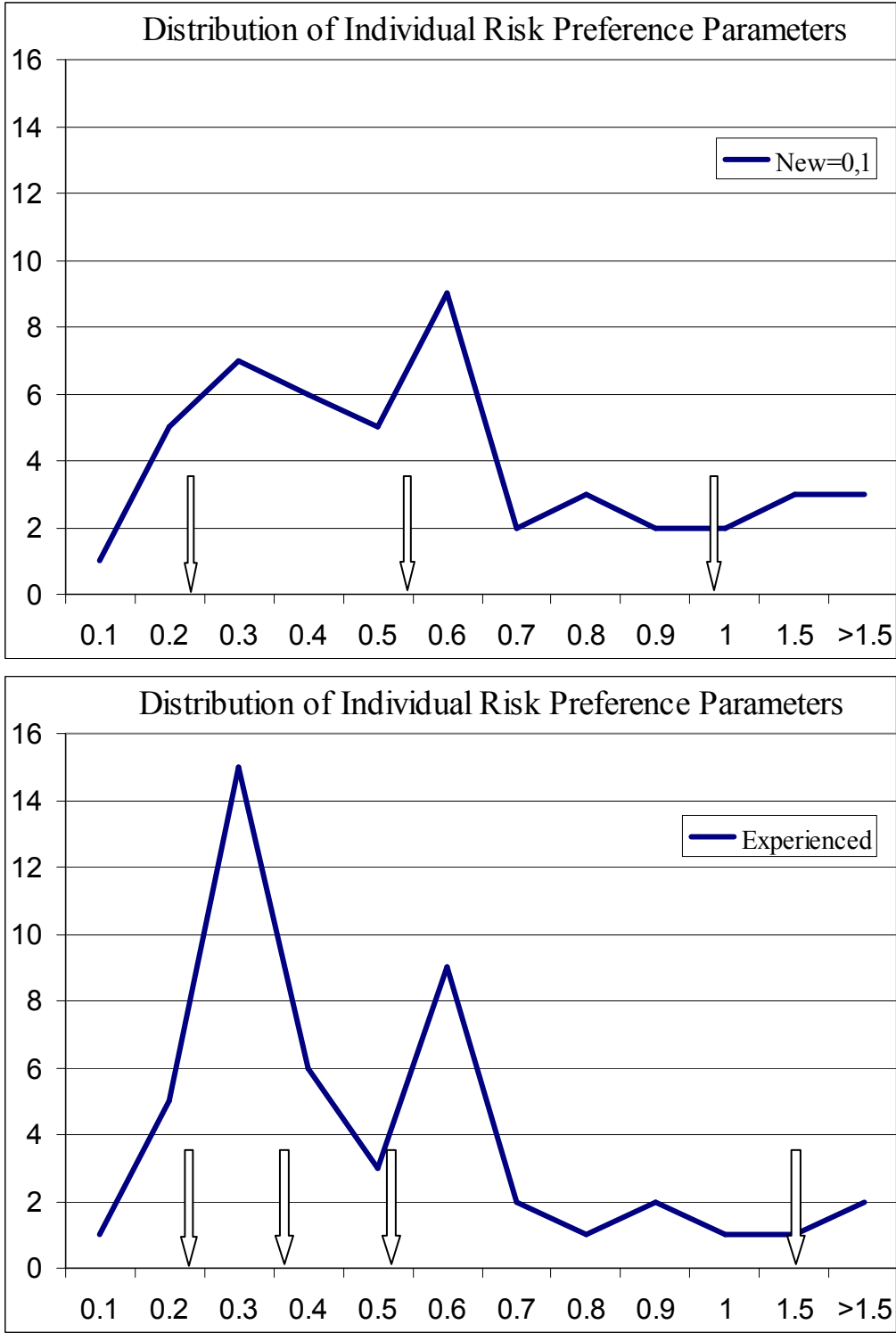


Figure 2. Distributions of Individual Risk Preference Parameters
 (arrows point to values of parameter estimates of EC algorithm segments)



Appendix.

EXPERIMENTAL INSTRUCTIONS (vSFI.4)

1. INTRODUCTION

This is an experiment in the economics of market decision making. Depending upon the decisions that you and other participants make, you have the opportunity to earn a considerable amount of money which will be paid to you in a check on a research account at the end of the experiment. Under no circumstances will you leave the experiment owing money to the experimenters. Your rights as a subject in an experiment have been explained to you in your sign-in document. Except as you might be directed in these instructions or by the experimenter, please do not talk or otherwise communicate with any other participant during the course of the experiment.

2. TODAY'S MARKETS

First, we must mention that from this point on, all dollar amounts, unless otherwise specified, are "experimental dollars" (E\$). At the end of the experiment, you will earn 25 U.S. cents for each one experimental dollar that you earn. Thus, if your payoff earnings say \$5.00 (E\$), you will actually earn \$1.25. If your earnings say \$8.00 (E\$), you will actually earn \$2.00 and so forth. The only amount that does not get adjusted by this exchange rate is your \$7.00 for showing up. That \$7.00 is already guaranteed in U.S. dollars. Therefore, from this point on, all costs, values, payments profits and so forth that we use will be *experimental dollars*.

Around this room you will see 16 "auction stations". At each station there is a different hypothetical item for sale by auction. Each item is known by a letter, which has no other meaning other than to identify each item. There is exactly one item for sale at each auction station.

The process for selling each item is as follows. At each station, there is a foam mug marked with the letter designation for the item for sale. You have been given your own pad of sticky notes. If you wish to make a bid to purchase that item, please take a sticky note and write on it **all** of the following four items:

- *The letter designation of the item, e.g. A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P
- *Your bidder number, e.g., 1, 2, 3, 4, 5, 6, 7, or 8
- *The time as shown on the clock
- *Your bid in experimental dollars and cents

Your sticky-note bid must have all four items on it to be a valid bid, and all decisions by the experimenters disqualifying incomplete bids are final. If you find that you submitted a bid with missing information, you can at any time submit a complete, second bid. It is recommended that you fold your completed bid form securely after you finish writing on it. This will provide a greater degree of privacy.

You may make bids on as many or as few of the 16 items as you like. You may submit more than one bid on an item. All auctions close at the same time (7 minutes elapsed time on the official timer). The winning bidder for each item is the highest bid in the coffee can when time has expired. The winning bidder pays his/her own bid (that is, the highest or winning bid) to the experimenter. If there is a tie for highest bid, one of the experimenters will break the tie using a random number table. Tie breaking decisions are final and non-negotiable. Different bidders may very well bid on different items. A single bidder may win zero, one, two, several, or even all of the items.

3. WHAT ARE MY VALUES FOR THE ITEMS? HOW ARE THEY CHOSEN?

First, it is important to note that each of you begins with a \$5.00 initial reserve simply for participating. This is in addition to your "show up" fee. All earnings from the auction markets are on top of this \$5.00 (remember, all amounts are in experimental dollars).

At the beginning of each auction period, you will be given a "payoff chart" so that you can keep track of your earnings. There is a column for each of the 16 items. The next row is labeled "my value for this item." The number in that row represents how much we will pay to you if you win the auction for that item. Some of these entries may be "zero."

If you win the auction for that item, then the following is how your earnings are calculated:

$$\begin{array}{r} \text{MY VALUE} \\ - \text{MY WINNING BID} \\ \hline \text{MY EARNINGS} \end{array}$$

If you do not win an item, your earnings at that auction station are zero.

Suppose, for example, your value for an item was \$24.00, and you won the auction with a bid of \$21.40. Then, your earnings would be: $\$24.00 - \$21.40 = \$2.60$ (remember, experimental dollars). (These numbers are for illustrative purposes only and have no meaning for the actual experiment).

Where do these values come from? That is a very good question. The values for each item generally differ from one bidder to another. That is why we used a random process to hand out the folders when you arrived. Furthermore, each bidder will generally have different values for different items, and different values for the same item between periods.

In choosing the values, we first decided for each item in each period how many bidders would have positive values. We used a random number process to choose, independently, a number of 2, 3, 4, 5, 6, 7, or 8 bidders having positive values. Each number was equally likely. Then, we used a similar random number process to decide which bidder IDs would be the bidders with the positive values. Each bidder ID was equally likely to be chosen for each item. Finally, for those bidder IDs with positive values on a particular item in a particular period, we drew a different value number from the set of number \$00.01, 00.02, ..., 19.99, 20.00. Each value was equally likely.

Notice that this means there will always be at least two bidders with positive values for each item.

In other words, consider the G object in round one. First, a random number process might tell us that three bidders will have positive values. Then, a second random number process might tell us that the three bidder IDs to receive the positive values for the G object are 1, 3, and 4. Finally, we would then draw three random values between 00.00 and 20.00, say \$0.78 for 1, \$2.08 for 3, and \$18.74 for 4. All other bidders would have value \$00.00 for G for round one. Different numbers, IDs, and values would be drawn for each object in each round. These numbers are hypothetical and have no relation to the actual numbers drawn, but they illustrate the process we went through for each object, and each bidder, in every round.

Your earnings in anyone period are the sum of all of the earnings on items you won. Your earnings (in experimental dollars) in the entire experiment are the sum of your earnings in each period.

4. CLOSING THOUGHTS

As described above the values for the items are likely to be different from one bidder to another, from one item to another for a particular bidder, and from **one** period to another. Recall that these are chosen independently, so that if you get several very low or very high values it means nothing for what might happen in the future.

All bidders are expected to cooperate with the rules of the experiment. By agreeing to participate, you agree **to** follow the rules, and you understand that anyone who does not do so may, at the discretion of the experimenter, be asked to leave the experiment with only the "show up" fee of \$7.00.

It is possible to lose money in an auction. This will occur if you bid higher than your value on an item and you win the auction. In such a case, $\text{YOUR VALUE} - \text{YOUR WINNING BID}$ would be negative. We do not stop anyone from doing this, however, if you bid less than or equal to your value, you will never be in a position to lose money. If you do make decisions such that you lose money, we will subtract it first from your accumulated earnings, and second from your \$5 initial reserve. Any bidder who loses so much money that he/she has eliminated their accumulated earnings and their \$5.00 initial reserve will be allowed to remain in the experiment only under a rule that they bid less than or equal to their value on each item from then on out.

CHANGE IN MARKET PRICE RULE

Until we announce otherwise, we will be using a different rule to determine the market price (the payment) for the winning bidder in the auctions, as follows:

The winning bidder for each item remains the highest bid in the foam cup when time has expired. However, the winning bidder pays to the experimenter *the highest bid submitted by a losing bidder*. If there is a tie for highest bid, one of the experimenters will break the tie using a random number table. Tie breaking decisions are final and non-negotiable.

All other aspects of the auctions remain exactly the same as in the original instructions.

It is still possible to lose money in an auction. This will occur if three things happen: i) you bid higher than your value on an item *and* ii) you win the auction *and* iii) your payment (the highest bid submitted by a losing bidder) is greater than your value. In such a case, $\text{YOUR VALUE} - \text{YOUR PAYMENT}$ would be negative. We do not stop anyone from doing this, however, if you bid less than or equal to your value, you will never be in a position to lose money. If you do make decisions such that you lose money, we will subtract it first from your accumulated earnings, and second from your \$5 initial reserve. Any bidder who loses so much money that he/she has eliminated their accumulated earnings and their \$5.00 initial reserve will be allowed to remain in the experiment only under a rule that they bid less than or equal to their value on each item from then on out.