Survival Auctions

John H. Kagel, Svetlana Pevnitskaya, and Lixin Ye*
The Ohio State University

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Abstract

Dynamic clock auctions with drop-out information typically yield outcomes closer to equilibrium predictions than do comparable sealed-bid auctions. However, clock auctions require congregating bidders for a fixed time interval, which has limited field applicability and introduces inefficiencies of its own given the time cost of congregating bidders. These inefficiencies can be removed by implementing a theoretically isomorphic survival auction – a multi-round sealed-bid auction with an information-revelation component, in which bidders are successively eliminated from one round of the auction to the next. We compare two different versions of a survival auction to Ausubel's version of a multi-unit demand, dynamic Vickrey auction with drop-out information provided. The s-stage survival auction is theoretically isomorphic to the Ausubel auction, with the two-stage survival mechanism yielding the same equilibrium outcome (via sincere bidding). The s-stage survival auction initially allocates items less efficiently than the Ausubel auction, however efficiency improves with bidder's experience.

Key words: survival auction, Vickrey auction, Ausubel auction, multi-unit demand auction.
JEL classifications: D44, D78, C92

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1 Introduction

Dynamic (clock) auctions with rivals drop-out information provided have consistently yielded closer conformity to equilibrium bidding strategies than comparable sealed-bid auctions for a variety of auction institutions and demand structures: uniform-price multi-unit demand auctions with and without synergies (Kagel and Levin, 2001, and 2002), single-unit, private-value auctions (Kagel, Harstad, and Levin, 1987), and single-unit common value auctions (Levin, Kagel, and Richard, 1996). In most, but not all cases, the same solution concept underlies the static and dynamic auctions so that the superior performance of the dynamic auction with drop-out information has been attributed to a transparency and simplicity that is simply lacking in most sealed-bid auctions (see, for example, Kagel and Levin, 2001). However, dynamic clock auctions suffer from a number of practical disadvantages that are likely to limit their applications to field settings as they require congregating all bidders for a fixed time interval. This also introduces inefficiencies of its own due to the time cost of congregating bidders. Alternatively, conducting quasi-dynamic clock auctions similar to those underlying the spectrum (air-wave) rights auctions (Cramton, 1997) can lead to exceedingly long auctions with an uncertain end point.

One promising practical alternative to the ascending-price clock auctions are "survival" auctions: multi-round sealed-bid auctions in which low bids are successively eliminated in every round with the low-bid price announced and needing to be met or exceeded in subsequent rounds. These survival auctions have been shown to be strategically equivalent to ascending-price clock auctions (Fujishima, McAdams, and Shoham, 1999). Survival auctions do not require congregating bidders for a fixed time interval and have a certain and swift end period. They also provide the drop-out information that is valuable for raising revenue in a number of auction settings, and has been shown to be the key factor in obtaining the closer to equilibrium outcomes in the ascending-price auctions. The present paper compares two different versions of the survival auction to a dynamic ascending-price auction format with drop-out information. We do so for the case of private value, Vickrey style auctions in which bidders demand multiple (up to two) units each. The ascending-price auction employs the Ausubel (2002) format, with drop-out information provided.

We compare two different survival auction mechanisms to the Ausubel auction: (1) An s-stage survival auction (where s is equal to the number of units initially demanded (or initially bid on) minus the number of units supplied) so that one active bid is dropped in each auction round until all units are allocated (Fujishima, McAdams, and Shoham, 1999) and (2) A two-stage survival format where everyone bids in the first stage, and only a limited number of high (stage-one) bids are permitted to bid in the second-stage (Perry, Wolfstetter and Zamir, 2000). We show that the strategic equivalence between the s-stage survival auction and the Ausubel auction still holds when bidders have multi-unit demands and Vickrey pricing rules are employed, and that all three auctions have the same equilibrium outcomes (via sincere bidding).

We find that the Ausubel auction achieves the highest level of sincere bidding, with the s-stage survival auction showing the most improvement over time so that by the last several auctions the level of sincere bidding approaches that of the Ausubel auctions. Deviations from sincere bidding in both the survival and two-stage auctions result primarily from bidding too low (below bidders' private values). This stands in marked contrast to the pervasive bidding above value found in comparable one-shot Vickrey auctions. The Ausubel auction provides the highest efficiency levels overall. The s-stage survival auctions show the most improvement over time, with efficiency levels approaching those of the Ausubel auction in the last several auction periods. These efficiency measures are compared to those resulting from purely random bidding and to a modified random bidding rule, as additional reference points against which to evaluate the different auction formats. We also compare seller revenue and bidder profits between auction institutions, and compared to the random bidding rules.

2 Theoretical Considerations

We consider an auction in which K indivisible identical objects are sold to n bidders, where $n > K$. Each bidder i ($i = 1, 2, \cdots, n$) demands up to two units of the good. Bidder i's valuation about object j is v_{ij}, $j = 1, 2$. v_{ij} is observable to bidder i but not observable to the other bidders. Ex ante, v_{ij}'s are independently drawn from uniform distribution with support $[0, \bar{v}]$. Three different auction formats are considered:

Ausubel auction with drop-out information provided: Bidders start out actively bidding on all units. A price "clock" starts at zero and increases continuously thereafter, with bidders deciding at what price to drop out of the bidding. Dropping out is irrevocable so a bidder can no longer bid on a unit he has dropped out on. Winning bidders pay the price at which they have "clinched" an item. Clinching works as follows: With K objects for sale, suppose at a given price, p_0, bidder i still demands two units, but the aggregate demand of all other bidders just dropped from K to $K-1$. Then in the language of team sports, bidder i has clinched winning an item no matter how the auction proceeds. As such bidder i is awarded one item at the clinching price, p_0. This process repeats itself with the supply reduced from

K to $K-1$ and with i's demand reduced by one unit. In this way the auction sequentially implements the Vickrey rule that each bidder pays the amount of the kth highest rejected bid, other than her own, for the kth object won. During the auction process, all drop-out prices are publicly reported when they occur, along with units clinched and the prices at which they were clinched. Ausubel (2002) shows that sincere bidding is the unique equilibrium surviving iterated elimination of (weakly) dominated strategies.

S-stage survival auction: The auction begins with each bidder submitting a vector of bids for both units. Bids from all subjects are ranked from highest to lowest with the low bid announced and the bid on that unit excluded in subsequent rounds. The lowest bid also becomes the minimal bid required for the following round. The auction proceeds in this way dropping the low bid in each round and determining who wins items, and the price paid for those items, using the clinching rules described for the Ausubel auction. (Clinched units and the prices paid for these units are announced as well.) This process repeats itself until all the items have been clinched, which takes exactly s stages, where s is equal to the number of units initially bid on minus the number of units supplied. For example, with 4 bidders bidding for 2 objects, it takes 6 stages ($s = 2 \times 4 - 2 = 6$) to complete the auction. In what follows we refer to the s-stage survival auction as *the survival auction*.

Two-stage survival auction: This is the same as the multi-stage survival auction described above except that it proceeds in two rather than multiple stages. That is, each auction consists of two rounds with only m high bids being active in the second round, where m is the minimal integer number such that clinching is not possible in the first round. For example, with 4 bidders bidding for 2 objects, setting $m = 4$ assures that there will be no clinching in the second stage; similarly, in the case with 4 bidders bidding for 3 objects, it can be verified that $m = 5$. The $2n - m$ low bids dropped in the first stage are revealed to the surviving bidders in the second stage, with all bids in the second stage required to be greater than or equal to the highest of the dropped bids from stage 1. In what follows we will refer to the two-stage survival auction as *the two-stage auction*.

The survival auction was first analyzed in Fujishima, McAdams, and Shoham (1999), who show that an ascending-price auction (aka Japanese clock auction in single-unit case; see Milgrom and Weber, 1982) and a survival auction are strategically equivalent for both single-unit and multi-unit object auctions.

In our case, the auction format under consideration involves bidders demanding multiple units and adopts Ausubel's clinching rule as the particular allocation and pricing rule. Following the basic arguments in Fujishima, McAdams, and Shoham, the strategic equivalence between a survival auction and an Ausubel auction can also be established.

Proposition 1 *The survival auction and the Ausubel auction are strategically equivalent.*

Proof: See Appendix.

Strategic equivalence is the strongest possible formal relationship between two mechanisms. Proposition 1 thus implies that survival auction and Ausubel auction are outcome equivalent.[1] Moreover, their equilibria coincide. Ausubel (2002) shows that sincere bidding by all bidders is the unique outcome of iterated elimination of weakly dominated strategies in the Ausubel auction with drop-out information provided. In view of the proposition above, we have the following corollary:

Corollary: *Sincere bidding in each round is the unique outcome of iterated elimination of weakly dominated strategies in both survival auction and Ausubel auction.*

Sincere bidding is also the unique outcome of iterated elimination of weakly dominated strategies in the two-stage survival auction.

Proposition 2 *A two-stage survival auction is (Nash) outcome equivalent to an Ausubel auction.*

Proof: See Appendix.

3 Experimental Design and Procedures

Each experimental session had two or more markets operating simultaneously, with subjects randomly reassigned to new markets in each auction period. There were four bidders in each market, with each bidder demanding two units. Bidders' demands are weakly decreasing, with two independent draws from a uniform distribution with support $[0, \$7.50]$ in each auction (with new draws in each auction period). Each auction employed an ABA design with supply (K) equal to 2 units in the first 12 auctions, $K = 3$ in the next 12, and $K = 2$ in the last 12 auctions.[2] Each auction began with two dry runs with $K = 2$.

[1]Two auctions are outcome equivalent if they possess (Nash) equilibria in which the items are allocated to the same set of bidders at the same set of prices.

[2]In one of the two-stage auctions there were 13 K=3 rounds. Data for the extra K = 3 auction have been dropped from the analysis.

Table 1 shows the number of sessions under each auction format along with the total number of experimental subjects.

Table 1: Experimental Treatments

Institution	Number of sessions	Number of subjects
Survival	2	20
Two stage	2	20
Ausubel	1	16[3]

All of the *Ausubel auctions* employed a "digital" price clock with a price increment of $0.25 every 3 seconds.[4] During the auction, the current price of the item, the number of items for sale, and the number of units actively bid on were posted on each bidder's screen at all times. Drop-out and clinching prices were also reported on all bidders' screens as they occurred (with these different prices clearly distinguished). When a bidder clinched an item the clinching price was automatically recorded on her computer screen just below the value of the item, with the profits earned for that item reported just below this.

In the *survival auctions* bids were submitted in each round with all but the unit with the lowest bid continuing to be actively bid on in the next round. After every round the bid on the unit that was dropped that round was reported along with drop-out bids on all units from previous rounds. Active bidders resubmitted their bids in every round with the restriction that bids in later rounds must be greater than or equal to the drop-out bid of the previous round.[5]

In *two-stage auctions,* m active bids survive from the first-stage bidding. The $2n - m$ low bids are reported to subjects after stage 1.[6] The number of second-stage bids, m, was set so that clinching was not possible in the first round ($m = 4$ for $K = 2$ and $m = 5$ for $K = 3$).

Following completion of all auctions all dropout prices and valuations were reported back to subjects, with dropout prices ranked from highest to lowest, and with own bids clearly distinguished from rivals.

[3]There were 8 subjects in the last $K = 2$ set of auctions as the time period we had recruited subjects for required that we permit those who needed to leave to do so for the last $K = 2$ treatment.

[4]Dropouts occurring within a given tick of the price clock were counted as having dropped out at the same price but with the drop-out order determined by when the file server recorded the drop-out.

[5]In case of ties for the low bid all tied bids were dropped. In case of ties for clinching, the computer randomly determined who earned a unit.

[6]In case of ties, the rank of the bids was determined randonmly. Then to ensure m active bids in the second stage, $2n - m$ low bids were dropped. In case of ties for clinching, the computer randomly determined who earned a unit.

Further, the same clinching metaphor employed in the Ausubel auctions was used to describe who earned items and the prices paid in both the survival and two-stage auctions.

Instructions were read out loud to subjects, with copies for them to follow along with as well. The instructions also included examples of how the pricing rules worked.[7]

4 Results

Table 2: Frequency of Sincere Bidding
(Standard errors of the mean in parenthesis. Differences from survival auctions in bold.)

	Higher Value Unit			Lower Valued Unit		
	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
K = 2	0.246 (0.063)	0.296 (0.073) **-0.050**	0.828 (0.052) **-0.582****	0.375 (0.067)	0.292 (0.056) **0.083**	0.761 (0.077) **-0.386****
K = 3	0.425 (0.078)	0.342 (0.085) **0.083**	0.833 (0.042) **-0.408****	0.567 (0.074)	0.417 (0.076) **0.150**	0.676 (0.083) **-0.109**
K = 2	0.600 (0.084)	0.396 (0.085) **0.204^{+}**	0.783 (0.079) **-0.183**	0.658 (0.069)	0.429 (0.065) **0.229****	0.556 (0.135) **0.102**

\+ Significantly different from 0 at the 10% level
** Significantly different from 0 at the 1% level

Bid Patterns: Table 2 reports the frequency of sincere bidding between the three auction procedures. The unit of observation employed is the mean frequency with which individual bidders were bidding sincerely computed over all 12 auctions for each value of K. Mann-Whitney tests are used to check for significant differences between auction institutions. Further, since 25¢ bid increments were employed in the clock auctions, bids are counted as sincere in the clock auctions if the dropout occurred at the clock tick just below the actual value or just above the actual value.[8] To give the sealed-bid auctions the same flexibility, a bid is counted as sincere if it occurred within plus or minus 12.5¢ of the actual value.[9]

[7] See http://www.econ.ohio-state.edu/lixinye/Experiment/Survival/Instruction/

[8] For example, suppose the value was $4.12, the bid would be counted as sincere if the drop occurred at 4.00 or 4.25.

[9] Effectively, 13¢ of the actual value. Winning bids are censored in the Ausubel auction but not in the survival auctions.

For the first $K = 2$ auctions, the frequency of sincere bidding is substantially (and significantly) lower in the survival auctions than in the Ausubel auctions for both high and low valued units. Further, there is little to distinguish between the survival and two-stage auctions on this account. However, there is substantial growth in the frequency of sincere bidding in the survival auctions for both units, and some deterioration, particularly for the lower valued unit, in the Ausubel auctions so that by the last $K = 2$ treatment there are no longer any significant differences between the two treatments. There is much more modest improvement in sincere bidding in the two-stage auctions than the survival auctions, so that by the last $K = 2$ auctions, there are statistically and economically meaningful differences in the frequency of sincere bidding between the survival auctions and the two-stage auctions. Finally, there is significantly less sincere bidding in the two-stage auctions than the Ausubel auctions for all but the lower valued unit in the last $K = 2$ treatment.

Conclusion 1 *There is substantially more sincere bidding to begin with in the Ausubel auctions than in either the survival auctions or the two-stage auctions. These differences persist for the two-stage auctions, but are gradually eliminated for the survival auctions.*

Accounting for this by dropping winning bids from the calculations for the survival auctions has no material effect on the results reported.

Table 3: Comparison of Bid Patterns Between Auction Mechanisms
(Frequencies with standard errors of the mean in parenthesis.
Differences from Survival auctions in bold.)

	Higher Value Unit			Lower Valued Unit		
K=2	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
Won and earned negative profits	0.021 (0.015)	0.046 (0.027) **-0.025**	0.009 (0.047) **0.012**	0.194 (0.125)	0.286 (0.184) **-0.092**	0.125 (0.210) **0.069**
Bid > v and not win	0.047 (0.025)	0.127 (0.052) **-0.080**	0.050 (0.112) **-.003**	0.119 (0.047)	0.187 (0.063) **-0.068**	0.180 (0.134) **-0.061**
Bid < v	0.648 (0.078)	0.570 (0.084) **0.078**	0.122 (0.089) **0.526****	0.498 (0.079)	0.510 (0.073) **-0.012**	0.060 (0.079) **0.438****
K=3						
Won and earned negative profits	0.011 (0.008)	0.027 (0.013) **-0.016**	0.000 (0.000) **0.011**	0.159 (0.077)	0.143 (0.082) **0.016**	0.103 (0.148) **0.056**
Bid > v and not win	0.158 (0.058)	0.033 (0.019) **0.125**+	0.069 (0.101) **0.089**	0.190 (0.068)	0.123 (0.053) **0.067**	0.294 (0.142) **-0.104**
Bid < v	0.325 (0.072)	0.616 (0.086) **-0.291***	0.098 (0.094) **0.227***	0.227 (0.063)	0.453 (0.074) **-0.226***	0.031 (0.055) **0.196****
K=2						
Won and earned negative profit	0.029 (0.020)	0.045 (0.022) **-0.016**	0.000 (0.000) **0.029**	0.000 (0.000)	0.167 (0.105) **-0.167**	0.250 (0.421) **-0.250**+
Bid > v and not win	0.053 (0.027)	0.127 (0.059) **-0.074**	0.139 (0.174) **-0.086**	0.134 (0.053)	0.164 (0.063) **-0.030**	0.400 (0.215) **-0.266***
Bid < v	0.379 (0.082)	0.479 (0.094) **-0.100**	0.078 (0.124) **0.310***	0.200 (0.052)	0.389 (0.074) **-0.189**+	0.044 (0.106) **0.156***

\+ Significantly different from 0 at the 10% level
* Significantly different from 0 at the 5% level
** Significantly different from 0 at the 1% level

Table 3 reports the pattern of deviations from sincere bidding between auction institutions. For both the survival and two-stage auctions the frequency of winning items and earning negative profits is quite low, comparable to the results reported for the Ausubel auctions. (Note, this measure for lower valued units is deceptively high as typically only a few subjects actually win a second, lower valued, unit, and our measure drops subjects who have not won any lower valued units.[10]) What Table 3

[10]This follows form unsing subject averages as the unit of observation. For example, in the last $K = 2$ treamtent for

shows is that the primary source of deviations from sincere bidding for both the survival and two-stage auctions is bidding *below* value. This holds to the point that there is significantly more bidding below value in both the survival and two-stage auctions throughout compared to the Ausubel auctions. What differentiates the survival and two-stage auctions on this score is that after the first $K = 2$ treatment, there is a substantial reduction in the frequency of bidding below value in the survival auctions, with much more modest improvement in the two-stage auctions. This is consistent with the changes in sincere bidding reported in Table 2.

Conclusion 2 *Deviations from sincere bidding for both the two-stage and survival auctions result primarily from bidding below value, involving opportunity costs rather than out-of-pocket losses.*

What is perhaps most striking about the underbidding in the two-stage and survival auctions is the contrast to the pattern found in the standard (one-stage) sealed-bid Vickrey auctions, which always involves bidding *above* value (Kagel and Levin, 1993; Kagel, Kinross, and Levin, 2003). These differences raise a number of obvious questions. For example, in a one-shot multi-unit demand Vickrey auction with the same parameters and procedures as those employed here (Kagel, Kinross, and Levin, 2004), the frequency of bidding above value averaged over 50% for all values of K (and often well 50%) compared to a maximum frequency of under 20% reported here for the survivor and two-stage auctions.

These results pose two important questions. First, why there are differences in the bid patterns between the standard, static Vickrey auction and the two-stage and survival auctions? Second, why the improvement in performance under the survival auctions as opposed to the much slower adjustments in the two-stage auctions?

A number of contributing factors would appear to be at work here. First, in a multi-unit demand context, characterizing the auction mechanism in terms of the clinching rules appears to play a significant role in getting subject not to bid above their value. We know this because we have implemented dynamic Vickrey auctions (without any dropout information) using the same terminology as in the sealed-bid auctions compared to using the clinching terminology employed here. Using the sealed-bid terminology subjects deviate from sincere bidding by bidding above value. In contrast, using the clinching terminology subjects typically deviate from sincere bidding by bidding below their values (Kagel, Kinross, and Levin, 2003). That is, there appears to be a clear framing effect as a consequence of the language employed in the instructions. Both the two-stage and survival auctions used the

the Ausubel auctions we have two subjects who actually won a low valued unit.

clinching terminology here. So this provides at least a partial explanation for the different bid patterns between the typical one-shot Vickrey auction and the two-stage and survival auctions.

With overbidding largely eliminated, subjects have only one type of error they can make - underbidding. With this in mind there are a number of differences between the auction institutions studied here with respect to the repeated nature of the choices subjects make. In the Ausubel auctions they must repeatedly decide (with each tick of the clock) to stay in or to drop out. In the survival auctions they must decide in each round how much to bid relative to their value. In the two-stage auction they make this decision twice. As such, subjects have, in effect, much more experience with the auction in the continuous clock case, an intermediate level of experience in the case of the survival auctions, and minimal experience in the two-stage auction. Thus, to the extent that subjects are learning from experience, we should see the quickest convergence to equilibrium for the Ausubel auctions, followed by the survival auctions, with the two-stage auctions showing the least amount of learning. This is exactly what we see in the data.[11]

[11]We find some evidence for subjects learning to not bid below value within the Ausubel auctions. Looking at the first $K = 2$ treatment, and breaking the data up into the first 6 auctions versus the last 6 auctions, we find underbidding freqeuncies of 14.3% (0.063) versus 6.9% (0.045) for the high valued unit (with standard errors of the mean are in parentheses) and 5.2% (0.025) versus 6.9% (0.030) for the low valued unit. Although a within subject Mann-Whitney (sign test) does not reject a null hypothesis of no difference at conventional levels, the difference with respect to the high valued unit is suggestive ($p < .15$ using a one-tailed test). For data on learning within dynamic clock auctions but in a different context see Kagel and Levin (2001).

Table 4: Comparisons of Efficiency, Profits and Revenue
(Standard errors of the mean in parentheses. Differences from Survival auctions in bold.)

	Efficiency			Revenue (difference from sincere bidding)			Profits (difference from sincere bidding)		
	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel	Survival	Two-stage	Ausubel
K = 2	93.8% (1.33)	93.1% (1.72) **-0.70%**	98.3% (0.77) **-4.50%****	-0.951 (0.227)	-1.085 (0.249) **0.134**	-0.211 (0.101) **-0.740****	0.110 (0.314)	0.209 (0.373) **-0.099**	0.018 (0.138) **0.092***
K = 3	97.4% (0.61)	94.9% (0.96) **2.50%***	98.8% (0.59) **-1.40%****	-0.391 (0.208)	-2.071 (0.257) **1.680****	-0.134 (0.089) **-0.257**	-0.066 (0.250)	1.217 (0.313) **-1.283****	-0.061 (0.114) **-0.005**
K = 2	98.7% (0.41)	95.2% (1.05) **3.50%+**	98.8% (1.24) **-0.01%****	-0.325 (0.090)	-0.988 (0.188) **0.663****	0.154 (0.098) **-0.479****	0.164 (0.108)	0.382 (0.234) **-0.220**	-0.316 (0.228) **-0.480***
Random[a]	60.9% [62.5%]			-0.32 [0.36]			-4.63 [-6.71]		
Random Survival[a]	76.6% [77.5%]			0.05 [0.19]			-3.01 [-4.05]		
Modified Random[a]	94.4% [95.0%]			0.56 [0.01]			-1.24 [-0.82]		

[a] Simulations for K = 3 are in brackets; K = 2 not in brackets.
[+] Significantly different from 0 at the 10% level.
[*] Significantly different from 0 at the 5% level.
[**] Significantly different from 0 at the 1% level.

Efficiency, profits and revenue: Table 4 provides data on efficiency, profits and revenue between the three auctions mechanisms. Efficiency is measured in the usual way - the sum of the K winning valuations divided by the sum of K highest valuations. Since in each auction valuations are drawn randomly, we report revenue and profits in terms of deviations from the equilibrium prediction. In all cases the unit of observation is the individual auction market. For comparative purposes we also report efficiency, revenue and profits based on several simulated "naive" bidding models. The first benchmark involves totally random bidding with bids based on random draws from the uniform distribution over the interval of possible values, i.e. subjects do not take into account their own valuation at all. [12] The second benchmark ("random survival") accounts for the fact that bids in each successive round had to

[12] There are two ways to do this. In what is reported the higher of the two draws is the bid for the higher valued unit and the lower of the two draws to be the bid for the lower valued unit. We have also done simulations for "restricted naive bidding" when the bid for the higher valued unit, b_h, is a draw from uniform distribution on $[\underline{v}, \bar{v}]$ and the bid for lower valued unit, b_l, is a draw from uniform distribution on $[\underline{v}, b_h]$. The effect of this is to raise efficiency a bit relative to the totally random bidding reported.

be higher than the bid that was dropped in the previous round. After the first round this restriction truncates the lower bound of the interval from which bids can be drawn. As can be seen, this restriction improves efficiency somewhat. In the the third benchmark, called modified random bidding, bids are drawn from the interval: $[v_i - \min\{v_i - 0, \$7.50 - v_i\}, v_i + \min\{v_i - 0, \$7.50 - v_i\}]$, where v_i is the bidder $i's$ valuation. In other words, bids are random draws from an interval with mean v_i and with a range equal to twice the distance between v_i and the closest bound of the support $[0, 7.50]$ from which valuations are drawn.[13] This leads to substantial improvements in efficiency - to over the 90% level. Using these bidding rules we ran 100 simulations of each auction computing mean values for each set of 12 auctions. These naive bidding models provide a benchmark for the value added due to "smart" human bidding.

The Ausubel auction generates the highest efficiency of all three mechanisms for all three auction sets. However, after the first $K = 2$ treatment the survival auction comes quite close, with essentially no difference in average efficiency between the two mechanisms in the last $K = 2$ treatment.[14] In contrast, the two-stage auctions show minimal improvement in efficiency, with the level significantly lower than under survival bidding for $K = 3$ and the final $K = 2$ treatment.

The results of the random bidding models provide a further benchmark against which to evaluate these efficiency measures. First, all mechanisms do substantially better than random bidding. Second, the two-stage auctions do not do much better than the modified random bidding rule, but both the Ausubel and survival auctions (the latter, after a bit of experience) do substantially better. The modified random bidding rule is, to our minds at least, still quite naive so that it is somewhat surprising that it yields such high efficiency levels.

Conclusion 3 *The Ausubel auctions start out with high levels of efficiency and stays that way throughout. The survival and two stage auctions start out with much lower efficiency levels, comparable to what the modified random bidding simulations suggest. Efficiency improves rather dramatically for the survival auctions, rivaling the levels found in the Ausubel auction in the last $K = 2$ treatment, but there is minimum improvement in efficiency for the two-stage auctions.*

Average revenue under the different auction formats is reported in the middle columns of Table 4. The first thing to note is that revenue is lower than predicted in both the survival and two-stage

[13] We did not simulate results for survival auctions using this measure because in cases where v_i is less than the drop-out price in the previous round, the interval is not well defined.

[14] The Mann-Whitney test yields a significant difference in the last $K = 2$ treatment as all but one of the Ausubel auctions had 100% efficiency, whereas a number of the survival auctions had less than 100% efficiency.

auctions. This results from the high frequency of bidding below value reported earlier. Revenue is closest to the equilibrium prediction across all three auction sets for the Ausubel auction. As a result average revenue is lower in the survival auctions than in the Ausubel auctions for all three auction sets, and is significantly less than the Ausubel auctions for both $K = 2$ treatments. The two-stage auctions have even lower revenue than the survival auctions with these differences statistically significantly in the $K = 3$ treatment and the last $K = 2$ treatment, as the survival auctions are converging to the equilibrium prediction, while the two-stage auctions show little improvement.

Average bidder profits are reported in the far most columns of Table 4. With the exception of the two-stage auction profits with $K = 3$, there is little in the way of economically meaningful differences in profits between the auction institutions. For the $K = 3$ treatment, the high frequency of underbidding found in the two-stage auctions results in substantially higher average profits than predicted, and substantially higher profits than under either of the alternative auction institutions.

Conclusion 4 *Average revenue in the survival and two-stage auctions tends to be below the level in the Ausubel auctions as a result of the higher frequency of bidding below value reported in the first two cases. This translates into higher bidder profits in the two-stage and survival auction compared to the Ausubel auctions.*

5 Summary and Conclusions

This paper looks at the applicability of survival auctions as an alternative to ascending-price, clock auctions which have been shown in a large variety of laboratory experiments to yield outcomes much closer to equilibrium predictions than sealed-bid auctions. We extend the survival auction mechanism originally suggested by Fujishima, McAdams and Shoham (1999) to multi-unit demand auctions that employ Vickrey allocation and pricing rules, and show that strategic equivalence holds between the survival auctions and ascending-price (Ausubel) auctions with drop-out information. This implies efficient unit allocations via sincere bidding. Realization of this theoretical prediction would favor using survival auctions over ascending-price auctions as the former have quick and predictable termination times, do not require bidder congregation, and have an information-revelation component.

We find that the survival auctions have excellent results, in terms of allocative efficiency, compared to the benchmark Ausubel auctions following an initial learning phase in which they do substantially worse than the Ausubel auctions. The survival auctions do significantly better than the static Vickrey auctions after an initial learning phase as well (see Kagel, Kinross, and Levin, 2004, for static Vickrey auction results using a design comparable to the one employed here). In contrast, a two-stage version of the survival auction suggested by Perry, Wolfstetter and Zamir (2000) does not do nearly as well as the survival or Ausubel auctions even after bidders have gained substantial experience with the mechanism.

One surprising behavioral result from the present experiment is that subjects tend to deviate from sincere bidding by bidding below their value as opposed to bidding above their value as in the single-round Vickrey auction. We attribute this to the use of the clinching terminology, which is quite natural for the quasi-dynamic nature of the survival and two-stage auctions, as opposed to the static explanation of the price and allocation rules that is natural to employ in the single-round Vickrey auctions. The data also indicate that the repeated nature of decisions made in the survivor auctions help to reduce the bidding below value reported, so that bids are converging on the sincere bidding predicted as bidders gain experience with the mechanism.

One skeptical response to the differences between mechanisms reported here is that all of this does not matter for "real world" bidders who are sophisticated and need only be told the logic underlying sincere bidding under any of the three mechanisms. There are several possible responses to this criticism. First, this view is far from universal as the debates leading up to the design of the FCC spectrum auctions show.[15] Second, the results reported here shift the burden of proof from those who believe that the details of the mechanism do not matter to "sophisticated" bidders to demonstrate that their view is correct. Third, Rutstrom (1998) has conducted an experiment comparing an English clock auction to a second-price sealed-bid auction in a single-unit private-value auction in which she went to great pains to explain the dominant bidding strategy.[16] The result was the typical pattern reported in the lab in cases where subjects are not offered any explanation for the dominant bidding strategy – prices were significantly higher in the second-price auctions. This indicates that, at a minimum, a more dynamic mechanism is more likely to continue to produce closer to equilibrium outcomes even when subjects are tutored on the correct bidding strategy.

The results reported here show promise for survival auctions as a viable alternative to ascending-price auctions in terms of generating desirable equilibrium outcomes, while having a number of prefer-

[15] For example, in their comments to the Federal Communications Commission describing the multi-unit Vickrey auction Nalebuff and Bulow (1993) write (p. 29): "However, experience has shown that even Ph. D. students have trouble understanding the above description [of the dominant bidding strategy] ... The problem is that if people do not understand the payment rules of the auction then we do not have confidence that the end result will be efficient."

[16] The auctioned item was a box of chocolates which is, presumptively, strictly private value or very close to it.

able institutional characteristics compared to ascending-price auctions. Future research should more thoroughly explore the properties of these survival auctions, particularly in the case of common value auctions, or auctions with affiliated private values, where the information aggregation inherent in revealing drop-out prices is predicted to raise revenue for sophisticated/experienced bidders and to reduce the incidence of the winner's curse for naive/inexperienced bidders (Levin, Kagel and Richard, 1996).

References

[1] Ausubel, Lawrence M. (2002): "An Efficient Ascending-Bid Auction for Multiple Objects," *Forthcoming, American Economic Review.*

[2] Cramton, Peter (1997): "The FCC Spectrum Auctions: An Early Assessment," *Journal of Economics and management Strategy*, 6:3, pp. 431-495.

[3] Nalebuff, Barry J. and Bulow, Jeremy I. (1993): "Designing the PCS Auction," Comment to the FCC on behalf of Bell Atlantic.

[4] Fujishima, Yuzo, David McAdams, and Yoav Shoham (1999): "Speeding Up Ascending-Bid Auctions," *Proceedings, Int. Journal Conf. in Artificial Intelligence.*

[5] Kagel, John H., Harstad, Ronald M., and Dan Levin (1987): "Information Impact and Allocation Rules in Auctions with Affiliated Private Values: A Laboratory Study," *Econometrica*, 55, pp. 1275-1304.

[6] Kagel, John H., and Dan Levin (2001): "Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions," *Econometrica*, 69, pp. 413-454.

[7] Kagel, John H. and Dan Levin (2002): "Multi-Unit Demand Auctions with Synergies: Behavior in Sealed-Bid versus Ascending-Bid Uniform Price Auctions," forthcoming *Games and Economic Behavior.*

[8] Kagel, John H., Scott Kinross, and Dan Levin (2003): "Implementing Efficient Multi-Object Auction Institutions: An Experimental Study of the Performance of Boundedly Rational Agents," *OSU Working Paper.*

[9] Levin, Dan, Kagel, John H., and Jean-Francois Richard (1996): "Revenue Effects and Information Processing in English Common Value Auctions," *American Economic Review,* 86, pp. 442-60.

[10] Milgrom, P. and Weber, R. (1982): "A Theory of Auctions and Competitive Bidding," *Econometrica*, 50, pp. 1082-1122.

[11] Perry, Motty, Elmar Wolfstetter and Shmuel Zamir (2000): "A Sealed-Bid Auction That Matches the English Auction," *Games and Economic Behavior* 33, pp. 265-273.

[12] Rutstrom, E. Elisabet (1998): "Homegrown Values and Incentive Compatibile Auction Design," *International Journal of Game Theory*, 27, pp.427-41.

Appendix

Proof of Proposition 1: Two auctions are strategically equivalent if there exists an isomorphism between their strategy spaces which preserves payoffs. Basically we can show that there exists an identity mapping between the information sets and their precedence relations in the two auctions, which leads to an identity mapping between the strategy spaces. We then verify that the (one-to-one) corresponding strategies induce the same payoffs. Following arguments paralleling those in Fujishima, McAdams, and Shoham (1999), we can proceed in three steps:

(1) In the survival auction, the new information that bidder i possesses in between round t and $t+1$, if he survives round t (i.e., if he is still active on at least one item), is the losing bid information and clinching information in round t. Losing bid information consists of the identity of the bidder who lost a bid, the unit on which the bid lost, and the bid amount. Thus the losing bid information in between round t and round $t+1$ can be described by a $3t$-dimensional vector. The clinching information can also be described by a $3t$-dimensional vector, consisting of the identity of the bidder who clinched a unit, the unit being clinched, and the price at which the unit was clinched (if no clinching occurs at round t, then all entries are filled by, say, letter "N "). Therefore, each surviving bidder's decision points in the $t+1$st round can be represented by a $6t$-dimensional vector. In the Ausubel auction, the new information that bidder i possesses in between round t and $t+1$, if he still stays in the auction (not dropping from all items), is the drop-out information and clinching information in round t. The drop-out information consists of the identity of the bidder who dropped out, the item on which this bidder dropped out, and the drop-out price. Thus the losing bid information in between round t and $t+1$ can be described by a $3t$-dimensional vector. Similarly, the clinching information can also be described by $3t$-dimensional vector. Therefore the isomorphism of decision point sets between two auctions is the identity mapping.

(2) In the survival auction, each bidder in the $(t+1)$st round, if still active, can make any new bid which is higher than the minimal bid in that round, i.e., the eliminated bid in the tth round. In the Ausubel auction, each bidder after the tth drop-out can decide to wait until any price higher than the last drop-out before being the next to drop out. Thus the feasible action sets are identical and the decision point precedence relation is preserved by the identity mapping.

(3) In both auctions, under the identity mapping, if the "same" terminal point is reached, then the actions must be the "same" at the "same" decision points – "same" in the sense that they are equivalent under the identity mapping. This implies the following: (a) the objects will be allocated to the same set of bidders, (b) the winners of the items will pay the same amounts, and (c) the information available

to all bidders at the end of the auction will be the same.

(1) and (2) imply that there exists an isomorphism (the identity mapping in this case) between the strategic sets in Ausubel and Survival auctions. (3) implies that the payoffs are preserved under this identity mapping. *Q.E.D.*

Proof of Proposition 2: We start with the second stage in a two-stage survival auction. Suppose the highest rejected bid in the first stage is b^*, then bidding $\max\{b^*, v_{ik}\}$ for $k \in \{1, 2\}$ is the weakly dominant strategy for each remaining bidder i who is still active on object k. All other strategies are weakly dominated. Now consider the first stage bidding. Given that bidders submit $\max\{b^*, v_{ik}\}$ in the second stage bidding (the outcome of one-round elimination of weakly dominated strategies), we claim that sincere bidding is the weakly dominant strategy in the first stage. (1) Bidding more than v_{ik} for bidder i on item k, say, bidding $v_{ik}^+ > v_{ik}$ is weakly dominated by bidding v_{ik}, as there is some positive probability that v_{ik}^+ will become binding for bidder i who ends up winning the item, in which event bidder i incurs loss. (2) Bidding less than v_{ik} for bidder i on item k, say, bidding $v_{ik}^- < v_{ik}$ is weakly dominated by bidding v_{ik}, as there is some positive probability that bidder i will be excluded from the second stage bidding, while she would make it to the second stage and make positive profit if she bid sincerely. All other strategies in the first stage are weakly dominated by sincere bidding, given that in the second stage each bidder bids $\max\{b^*, v_{ik}\}$. This shows that sincere bidding is the unique outcome of iterated elimination of weakly dominated strategies in the two-stage survival auction. *Q.E.D.*