ECO5423
Homework 2

1. Consider the model:

$$Y_i = \begin{cases} (1+\lambda X_i\beta)^{\frac{1}{\lambda}} + \varepsilon_i & \lambda \neq 0 \\ \exp(X_i\beta) + \varepsilon_i & \lambda = 0 \end{cases}$$

Explain how this model may be estimated by NLS.

2. In the latest of a long series of revisions to your dissertation prospectus, you estimate the following model:

$$W = \alpha + \beta T + \gamma J + \delta N + \theta S + \epsilon$$

where W is the log of the transfer fee, T is goals scored, J is the number of years of first-division experience, N is the number of international appearances, and S is shoe size. All of the estimated coefficients were statistically significant except for that of shoe size. A member of your dissertation committee, Professor Zuehlke, complains that your estimates are "hopelessly biased " because shoe size is an extraneous variable. He suggests that, at a minimum, you "toss that junk" and re-estimate your model. Unfortunately, this is impossible since all copies of your data were lost when, in a somewhat comatose state following your interminable duties at the economic help desk, you accidentally bumped into Professor Fournier as he hurried to class, knocking your laptop over the railing of the Bellamy atrium. How would you respond? Remember, you cannot re-estimate!

3. You are attempting to measure the “marginal value of floor space.” You estimate the regression model

$$SP = \alpha + \beta FLOOR + \epsilon$$

Unfortunately, the true model is

$$SP = \alpha + \beta FLOOR + \delta TRAV + \epsilon$$

where greater floor space and shorter travel time are both valued by buyers. That is, we expect $\beta > 0$ and $\delta < 0$. If newer neighborhoods with larger houses are in outlying areas, what is the likely direction of bias introduced by the omission of TRAV?

4. Consider the regression model

$$Y = \alpha + \beta X + \epsilon$$

where $E(\epsilon|X) \neq 0$. An instrument, Z, is available. This instrument satisfies $E(\epsilon|Z)=0$. Show that the variance of the IV estimator explodes as the correlation between X and Z approaches zero.

5. Consider the following "structural" system of equations.

$$Y_i = \beta Z_i + \gamma X_i + \epsilon_i$$

$$Z_i = \delta Y_i + \theta W_i + \eta_i$$

The "endogenous" variables, Y and Z, are determined simultaneously. The "exogenous" variables X and W are determined externally and assumed to be non-stochastic. The stochastic errors $(\epsilon_i, \eta_i) \sim iid(0, \Sigma)$ where

$$\Sigma = \begin{bmatrix} \sigma^2 & \kappa \\ \kappa & \tau^2 \end{bmatrix}$$

The "reduced form" equations are the solution for each endogenous variable in terms of the exogenous variables and stochastic errors alone.

a. Solve for the reduced form equation for Z by eliminating Y from the second equation.

b. Use the reduced for equation for Z to find the covariance of Z and ϵ.

c. Will OLS estimation of the first structural equation provide unbiased estimates?

d. Which of the variables in this system are potential "instruments" for Z?

6. Use proof by induction to show that

$$\sum_{t=1}^{T} t^2 = \frac{T(T+1)(2T+1)}{6}$$