Duration Dependence Testing for Speculative Bubbles

Yvette S. Harman and Thomas W. Zuehlke*

Abstract

Prior research has employed a number of methods to test for speculative bubbles in asset prices, including a method based on the concept of duration dependence. This study explores whether duration dependence tests for speculative bubbles are sensitive to specification decisions. Our results question the efficacy of using measures of duration dependence to test for speculative bubbles. In particular, we find that evidence of duration dependence is sensitive to the method of correcting for discrete observation of continuous duration, the use of value-weighted versus equal-weighted portfolios, and the use of monthly versus weekly runs of abnormal returns. (*JEL* C41, G12)

Introduction

The term *speculative bubble* is often used to describe persistent market overvaluation followed by market collapse. Bubbles arise when assets consistently sell at prices in excess of what is deemed an asset's fundamental value. Prior research has employed a number of methods to test for speculative bubbles in asset prices. One method, employed by McQueen and Thorley (1994), is based on the statistical theory of duration dependence. Their theoretical model suggests that if security prices contain bubbles, then runs of positive abnormal returns will exhibit negative duration dependence (decreasing hazard rates). That is, the conditional probability of a run ending, given its duration, is a decreasing function of the duration of the run.

The estimates reported by McQueen and Thorley (1994) are consistent with the presence of speculative bubbles in the New York Stock Exchange (NYSE). They find statistical evidence of negative duration dependence in runs of positive abnormal monthly returns for both equal-weighted and value weighted portfolios of NYSE-traded securities. In contrast, Chan, McQueen, and Thorley (1998) test for speculative bubbles in monthly and weekly returns from selected Asian stock markets and the Standard & Poor's 500 (S & P 500). Chan, et al. find no statistical evidence of duration dependence in returns to the S & P 500, and only one case of duration dependence consistent with rational speculative bubbles among the Asian markets.

In this study, we explore the sensitivity of duration dependence tests for speculative bubbles to a number of important specification decisions. To address this issue, we test returns to the NYSE and NYSE-AMEX portfolios using both continuous hazard models and discrete hazard models. Included among these models is the discrete log-logistic model of McQueen and Thorley (1994). Runs of positive and negative abnormal returns are constructed for both equally-weighted and value-weighed portfolios, on both a monthly and weekly basis.

Our results suggest that the estimated duration elasticity is sensitive to the method of correcting for discrete observation of continuous duration. We also find that the nature of duration dependence is sensitive to the choice of value-weighted versus equally-weighted portfolios, and to the construction of monthly versus weekly runs of abnormal returns. As such, our results raise questions about the use of hazard models to test for speculative bubbles.

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* Yvette S. Harman, Department of Finance, Miami University, Oxford, OH 45056 and Thomas W. Zuehlke, Department of Economics, Florida State University, Tallahassee, Florida 32306

Methodology

The theoretical model of rational speculative bubbles presented in McQueen and Thorley (1994) suggests that the bubble process leads to explosive price changes. As such, the bubble grows each period that it survives. As the bubble component grows, it begins to dominate the fundamental component, i.e., that portion of the stock price determined by the discounted value of the future cash flows. Negative abnormal returns become less likely and generally only occur when the bubble bursts. A long run of positive abnormal returns suggests the presence of a bubble and a decreased likelihood of negative abnormal returns. If securities prices contain bubbles, then runs of positive abnormal returns will exhibit negative duration dependence; i.e., the conditional probability of a run ending, given its duration, is a decreasing function of the duration of the run. Because bubbles cannot be negative, no such restriction can be inferred for runs of negative abnormal returns.[1]

Duration dependence is a characteristic of the hazard function for duration times.[2] If f(t) denotes the density function for duration times, and F(t) the corresponding distribution function, then the hazard function h(t) is defined as the conditional density function for duration of length t, given that duration is not less than t. That is, $h(t)=f(t)/[1-F(t)]$. The hazard function exhibits positive (negative) duration dependence if h(t) is increasing (decreasing) in t. The model of McQueen and Thorley (1994) predicts that the hazard function for a run of positive abnormal returns is a decreasing function of the length of the run.

Perhaps the most commonly used hazard model is the Weibull hazard. Specifically,

$$h(t) = \alpha(\beta+1)t^{\beta} \tag{1}$$

where $\alpha > 0$, $\beta > -1$, and $t > 0$. The parameter β is the duration elasticity of the hazard function. The Weibull hazard function is monotone in duration, and exhibits positive (negative) duration dependence if β is positive (negative). The log-likelihood function for a sequence of runs must be expressed in terms of the density function for duration (completed runs) and its distribution function (partial runs). Lancaster (1979) shows that the distribution function of the Weibull hazard specification is

$$F(t) = 1 - \exp(-\alpha t^{\beta+1}) \tag{2}$$

with corresponding density function

$$f(t) = \alpha(\beta+1)t^{\beta}[\exp(-\alpha t^{\beta+1})] \tag{3}$$

If J_i is a binary variable that indicates whether an observed duration, t_i, is complete or partial, then the log-likelihood function for a sequence of N runs is

$$\ln L(\alpha,\beta) = \sum_{i=1}^{N}\left\{J_i \ln[f(t_i)] + (1-J_i)\ln[1-F(t_i)]\right\} \tag{4}$$

While of relative insignificance in large samples, partial runs may occur at the beginning and end of the sample period. This specification was developed in an environment where duration was measured continuously, or at least, where the unit of time measurement was small relative to the magnitude of duration.[3]

Although securities markets trade continuously, the need to construct a sequence of abnormal returns from variables that are measured only at discrete points in time results in a sequence of runs that is measured discretely.[4] The precision with which duration is measured is limited by the unit of time (days, weeks, months, etc.) used to construct the sequence of abnormal returns. A run with an observed length of t_i

[1] Diba and Grossman (1987, 1988) contend that the model for speculative bubbles does not allow the existence of negative bubbles since the bubbles would grow more negative over time, yet stock prices cannot be negative.

[2] This terminology originated in survival analysis, where duration refers to length of life and the hazard is death.

[3] Crowder (1991) argues that there is generally little consequence to analyzing discretely measured durations as if continuous data. Many studies have taken this approach. For example, Sichel (1991) considers duration dependence in the NBER monthly business cycle chronology, and Cochran and DeFina (1995) consider duration dependence in stock market cycles, where duration of market expansions and contractions is measured in months.

[4] McDonald, McQueen, and Thorley (1995) provide Monte Carlo evidence that treating rounded duration data as continuous results in an upward bias in the estimated duration elasticity. The severity of the problem is contingent on the degree of rounding.

(an integer) could have been generated by any value of duration within an interval of plus or minus half a time unit about the observed value. Durations are essentially rounded to the nearest unit of time. The log-likelihood function for such a sample is

$$\ln L(\alpha,\beta) = \sum_{i=1}^{N}\left\{J_i \ln[F(t_i+0.5)-F(t_i-0.5)]+(1-J_i)\ln[1-F(t_i-0.5)]\right\} \tag{5}$$

McDonald, McQueen, and Thorley (1995) find this to be an effective approach to controlling for discrete observation of duration.

An alternative approach, used by McQueen and Thorley (1994), is to construct a discrete hazard model.[5] If g(t) denotes the discrete density function for duration and G(t) the corresponding distribution function,
then the log-likelihood function for a sequence of N runs is

$$\ln L(\alpha,\beta) = \sum_{i=1}^{N}\left\{J_i \ln[g(t_i)]+(1-J_i)\ln[1-G(t_i)]\right\} \tag{6}$$

where the discrete density and distribution functions for duration are related as

$$G(t_i) = \sum_{k=1}^{ti} g(k) \tag{7}$$

The relationship between the hazard function and the density function for completed duration must now be determined. If the hazard probability is defined as the probability that the completed duration equals t, given it is at least t, then for the discrete case we have h(t)=g(t)/[1-G(t-1)]. This implies that g(1)=h(1), since G(0)=0. This fact, and successive application of the law of conditional probability gives the density for completed duration as

$$g(k) = h(k)\Pi_{m=0}^{k-1}\left[1-h(m)\right] \tag{8}$$

for positive integer k, where h(0) is defined as zero.

All that is left is to specify the hazard function. A natural choice, for purposes of comparison, is the Weibull hazard function. An alternative choice, one made by McQueen and Thorley (1994), is the log-Logistic transformation

$$h(k) = \left\{1+\exp[-\alpha-\beta\ln(k)]\right\}^{-1} \tag{9}$$

This is also reasonable, since it makes the hazard probabilities monotone in duration and generates hazard probabilities on the unit interval. The discrete logistic hazard model exhibits positive (negative) duration dependence when β is positive (negative). It is important to note, however, that any number of hazard functions could be used in place of the Weibull or Logistic.

The theoretical model of speculative bubbles in McQueen and Thorley (1994) offers the following testable hypotheses: 1) runs of positive abnormal returns exhibit negative duration dependence, and 2) runs of negative abnormal returns exhibit no duration dependence. Hence, the null hypothesis of "speculative bubbles" involves testing: 1) the null hypothesis $\beta < 0$ against the alternative hypothesis $\beta \geq 0$ for runs of positive abnormal returns, and 2) the null hypothesis $\beta = 0$ against the alternative hypothesis $\beta \neq 0$ for runs of negative abnormal returns. In practice, McQueen and Thorley (1994) simply test the null hypothesis $\beta = 0$ against the alternative hypothesis $\beta \neq 0$ for both positive and negative runs. An estimate of β that is negative and significantly different than zero for positive runs, in conjunction with an insignificant estimate of β for negative runs, is considered evidence of speculative bubbles.[6]

[5] Sichel (1991) argues that although duration is often recorded as an integer, a discrete hazard model may not be appropriate because the underlying durations are actually continuous. He contends that the preferred approach is a continuous hazard model that accounts for the errors in measurement of duration.

[6] Note that, for a given level of significance, an estimate of the duration elasticity may be "significantly negative," while not "significantly different than zero." This does not occur at conventional significance levels for the estimates reported in this paper.

Data

The samples analyzed in this paper consist of firms with securities data available in the University of Chicago's Center for Research in Security Prices (CRSP) files. The securities are traded on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX). A sequence of monthly abnormal returns is constructed for both equal-weighted and value weighted portfolios of all NYSE stocks from 1927 through 1997. For purposes of comparison, a weekly sequence of abnormal returns is also constructed, using the equal-weighted and value weighted NYSE-AMEX indices for the period 1963 through 1997.

Construction of the monthly series is consistent with McQueen and Thorley (1994). Real returns are calculated by subtracting continuously compounded inflation rates from continuously compounded nominal returns.[7] A sequence of real abnormal returns is determined by the residuals from a regression of real returns on its first three lagged values, the term spread, and the dividend yield. Fama and French (1989) argue that the term spread and dividend yield are useful in predicting time-varying risk premia. Consistent with Fama and French (1989) and McQueen and Thorley (1994), the term spread is measured as the difference in yield-to-maturity between the Ibbotson Associates' AAA Corporate Bond Portfolio and the one-month Treasury bill. The dividend yield is the value weighted NYSE portfolio's dividend yield calculated by dividing the sum of the prior twelve monthly dividends by the current price. Both the term spread and the dividend yield are measured at the end of the prior period.

Construction of a sequence of abnormal returns for weekly data is limited by the absence of weekly measures of inflation, dividend yield, and term spread. Thus, we follow Chan, McQueen, and Thorley (1998), where runs of weekly abnormal returns are determined by the residuals from a fourth-order autoregressive model of weekly returns. Chan, et al., argue that an AR(4) model is preferable to imposing a common mean, because it enables one to control for short-term sources of autocorrelation, such as non-synchronous trading. Any remaining correlation is then attributed to price bubbles.

Once a time series for abnormal returns is constructed, the corresponding sequences of positive and negative runs can be determined. The length of the run is simply the number of consecutive periods of like sign in abnormal returns. As noted by McQueen and Thorley (1994), any evidence of speculative bubbles must be interpreted as conditional on the specification of the model used to generate the sequence of abnormal returns.

Results

The results in Table 1 are based upon the monthly sequence of abnormal returns for the 1927 through 1997 sample period. Estimates of the Continuous Weibull, Interval Weibull, Discrete Weibull, and Discrete Logistic hazard models are reported, for both equally-weighted and value-weighted portfolios of NYSE securities. Separate hazard models are estimated for runs of positive and negative abnormal returns.

Panel A of Table 1 reports estimates of the Continuous Weibull model (equations 2 through 4). The estimated duration elasticity is positive and significantly different than zero at the one percent level, for both positive and negative runs. This is true for both equally-weighted and value-weighted portfolios. These estimates strongly reject the hypothesis of rational speculative bubbles.

Panel B of Table 1 reports estimates of the Interval Weibull model (equations 2 and 5), which controls for the discrete observation of continuous durations. The results are qualitatively and quantitatively similar to those of Panel A. As with the Continuous Weibull model, the estimated duration elasticity is positive and significantly different than zero at the one percent level, for both positive and negative runs, and for both equally-weighted and value-weighted portfolios. The estimated duration elasticities are slightly smaller with the Interval Weibull model; a reduction of far less than a standard deviation in either case.

Panel C of Table 1 reports estimates of the Discrete Weibull model (equations 1, 6 through 8). This specification models duration as a discrete random variable, as opposed to discrete observation of a continuous random variable. For equally-weighted portfolios, the estimated duration elasticity is negative, but is not significantly different than zero at conventional levels, for both positive and negative runs of

[7] Continuously compounded inflation rates are constructed from the monthly inflation series available in the *Stocks, Bonds, Bills, and Inflation 1999 Yearbook* by Ibbotson Associates. This series is based on the Consumer Price Index without seasonal adjustment.

abnormal returns. For value-weighted portfolios, however, the estimated duration elasticity is negative and significantly different than zero at the 5 percent level for positive runs of abnormal returns, and is insignificant for negative runs of abnormal returns. Estimate of the Discrete Weibull model are consistent with the speculative bubbles hypothesis of McQueen and Thorley (1994) for value-weighted portfolios, but not for equally-weighted portfolios.

Table 1. Estimates for Monthly Abnormal Real Returns to NYSE Portfolios

Panel A: Continuous Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.288	0.404	0.305	0.600	0.293	0.341	0.313	0.596
(6.699)	(5.586)	(5.916)	(5.466)	(6.021)	(4.38)	(6.292)	(6.335)
Panel B: Interval Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.302	0.372	0.320	0.562	0.308	0.308	0.328	0.562
(6.662)	(5.055)	(5.799)	(4.944)	(6.006)	(3.903)	(6.189)	(5.794)
Panel C: Discrete Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.577	-0.140	0.599	-0.073	0.589	-0.167	0.597	-0.062
(2.092)	(-0.711)	(1.215)	(-0.192)	(7.333)	(-2.293)	(2.324)	(-0.284)
Panel D: Discrete Logistic							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
-0.013	-0.256	0.218	-0.150	-0.028	-0.313	0.241	-0.137
(-0.026)	(-0.730)	(0.262)	(-0.200)	(-0.286)	(-2.561)	(0.545)	(-0.296)

Notes: Asymptotic t-statistics in parenthesis.

Panel D of Table 1 reports estimates of the Discrete Logistic model (equations 6 through 9). This is the same model used by McQueen and Thorley (1994). It is similar to the Discrete Weibull model of Panel C, except that the log-Logistic transformation of is used to specify the hazard function. It is not surprising that the estimated duration elasticities are quantitatively similar to those of the Discrete Weibull model.[8] The point estimates are slightly lower, but within a standard deviation in each case. As with the Discrete Weibull model, the estimated duration elasticity is negative and insignificant for both positive and negative runs of abnormal returns from an equally-weighted portfolio.[9] For value-weighted portfolios, however, the estimated duration elasticity is negative and significantly different than zero at the 5 percent level for positive runs of abnormal returns, and is insignificant for negative runs of abnormal returns. As with the

[8] The estimated values of α are lower with the Discrete Logistic model than with the corresponding Discrete Weibull model. This is to be expected. Note that when $\beta=0$, the Weibull hazard function takes the value 0.5 for $\alpha=0.5$, while the Logistic hazard function takes the value 0.5 for $\alpha=0$. The estimates of α for these two specifications are not comparable.

[9] McQueen and Thorley (1994) find the estimated duration elasticity for positive runs from an equally weighted portfolio to be negative and significantly different than zero at the 5 percent level. This difference in results may be due to their use of a slightly shorter sample period and a likelihood ratio test statistic.

Discrete Weibull model, the results are consistent with the speculative bubbles hypothesis of McQueen and Thorley (1994) for value-weighted portfolios, but not for equally-weighted portfolios.

The results of Table 1 illustrate the sensitivity of the estimated duration elasticity to the method of correcting for discrete observation of duration and to the choice of value-weighted versus equally-weighted portfolios. The estimates of the Continuous and Interval Weibull models consistently yield evidence of positive duration dependence for runs of both positive and negative abnormal returns. This is true with both equal-weighted and value-weighted portfolios. In contrast, the estimates of the Discrete Weibull and Discrete Logistic models yield no evidence of duration dependence in either positive or negative runs of abnormal returns for equally-weighted portfolios. Yet when the same models are applied to value-weighted portfolios, the results are consistent with the speculative bubbles hypothesis of McQueen and Thorley (1994).

Given that they have the same underlying hazard function, it is interesting that the estimates of the Interval Weibull and Discrete Weibull differ with this data. The conclusion to be drawn is that the Interval and Discrete models are not necessarily good substitutes. Any similarities or differences in the estimates from these models are data specific. For example, when applied to the data set consisting of the duration of litigation in civil lawsuits examined in Fournier and Zuehlke (1996), all four models considered in this paper gave virtually identical results.

Table 2. Estimates for Weekly Abnormal Returns to NYSE-AMEX Portfolios

Panel A: Continuous Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.270	0.496	0.255	0.684	0.273	0.563	0.243	0.695
(9.112)	(7.841)	(9.684)	(10.749)	(9.868)	(10.047)	(9.017)	(9.379)
Panel B: Interval Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.283	0.466	0.265	0.660	0.284	0.535	0.252	0.674
(9.058)	(7.229)	(9.458)	(10.060)	(9.733)	(9.295)	(8.821)	(8.778)
Panel C: Discrete Weibull							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
0.524	-0.072	0.437	0.097	0.498	-0.011	0.412	0.121
(4.903)	(-0.692)	(3.492)	(0.646)	(1.281)	(-0.034)	(5.049)	(1.178)
Panel D: Discrete Logistic							
Equally-Weighted				*Value-Weighted*			
Positive Runs		*Negative Runs*		*Positive Runs*		*Negative Runs*	
α	β	α	β	α	β	α	β
-0.056	-0.129	-0.084	0.197	-0.028	-0.021	-0.152	0.235
(-0.300)	(-0.706)	(-0.266)	(0.615)	(-0.030)	(-0.034)	(-0.680)	(1.108)

Notes: Asymptotic t-statistics in parenthesis.

The sensitivity of the results in Table 1 to the method of correcting for discrete observation of continuous duration suggest that the estimates obtained from the monthly sequence of abnormal real returns might suffer from a time aggregation problem. In both this study and McQueen and Thorley (1994), over fifty percent of the runs have a completed duration of one month. When measured on a weekly basis, such runs vary in length anywhere between 2 to 6 weeks, yet when measured on a monthly basis, they all have the common length of 1 month. Much of the weekly variation in a sequence is washed out when measured on a monthly basis. A monthly sequence may provide insufficient variation in run length to effectively determine the duration elasticity.

The estimates reported in Table 2 are based on the weekly sequence of abnormal returns constructed from the NYSE-AMEX index over the period 1963-1997. The format of Table 2 is similar to that of Table 1. We report estimates of all four hazard models, for both positive and negative runs, and for both equally-weighted and value-weighted portfolios. It is interesting that the estimated duration elasticity is uniformly higher with weekly data. For the Continuous and Interval Weibull models, the estimated duration elasticity is positive and significantly different than zero for both positive and negative runs of abnormal returns. This is true for both the equal-weighted and value-weighted portfolios. As with monthly data, the rational speculative bubble hypothesis is strongly rejected.

With monthly data, the only evidence supporting speculative bubbles was found using the Discrete Weibull and Discrete Logistic models in conjunction with value-weighted portfolios. In contrast, neither of these models provides evidence of speculative bubbles when used with weekly data. For runs of positive abnormal returns, the estimated duration elasticity is negative, but is greatly reduced in absolute value relative to the estimates obtained with monthly data. For both equally-weighted and value-weighed portfolios, the estimated duration elasticity is not significantly different than zero. For runs of negative abnormal returns, the estimated duration elasticity is now positive, but as with monthly data, is not significantly different than zero. This result holds for both equal-weighted and value-weighted portfolios. Thus, there is no evidence of rational speculative bubbles in the weekly data.

Conclusion

Using a test of duration dependence with a discrete hazard model, McQueen and Thorley (1994) examine monthly returns to the NYSE portfolio for evidence of speculative bubbles. They argue that negative duration dependence in runs of positive abnormal returns is indicative of rational speculative bubbles. Chan, McQueen, and Thorley (1998) apply the same discrete hazard model to weekly returns on the S & P 500 portfolio and find no evidence of duration dependence.

In this study, we explore the sensitivity of duration dependence tests for speculative bubbles to several important specification decisions. In doing so, we find that our results raise some troublesome issues. Specifically, we find that the conclusions obtained are sensitive to the choice of sample period, the method of controlling for discrete observation of continuous duration, the use of equally-weighted versus value-weighted portfolios, and the use of weekly versus monthly returns. The sensitivity of the conclusions to so many specification choices calls into question the efficacy of using hazard models to test for speculative bubbles.

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