

**Applying an Alternative Test of Duration Dependence
to the Analysis of Speculative Bubbles**

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Abstract

This paper employs the generalized Weibull model of Mudholkar, Srivastava, and Kollia (1996) to examine the nature of speculative bubbles in security prices. This model is sufficiently flexible to allow changes in the direction of duration dependence. We find evidence to suggest that duration dependence is not monotonic. The estimated duration elasticity is initially positive, but becomes negative as the duration of the run of abnormal returns increases. The significant duration dependence found in runs of both positive and negative abnormal returns is inconsistent with the model of rational speculative bubbles proposed by McQueen and Thorley (1994).

Keywords

Duration dependence; rational speculative bubbles; generalized Weibull distribution

I. Introduction

Theoretical and empirical studies suggest speculative bubbles can lead to security prices that deviate substantially from their fundamental values. Shiller (1989) argues that when bubbles occur, price increases lead to successively larger increases. At some point, the price reaches a barrier and as there are no further price increases to sustain demand, the bubble bursts and the price drops sharply. Although researchers have historically considered speculative bubbles as evidence of investor irrationality, some research now suggests that rational price bubbles can occur in financial markets.

McQueen and Thorley (1994) provide a model of speculative bubbles that allows stock prices to differ from their fundamental values without assuming irrationality on the part of investors. Their model implies that if security prices contain bubbles, then runs of positive abnormal returns will exhibit negative duration dependence. That is, the longer the bubble persists, the lower the hazard probability of a crash. In contrast, the hazard probability for runs of negative abnormal returns will be independent of the duration of the run. McQueen and Thorley (1994, pg. 385) argue that this pattern of duration dependence is “indicative of the existence of bubbles since it cannot be the result of asymmetric or leptokurtic innovations in fundamentals alone.”

The conventional Weibull hazard model is a useful benchmark for studies of duration dependence. It imposes a linear relationship between the log-hazard function and log-duration. The Log-Logistic hazard model employed by McQueen and Thorley (1994) allows modest departures from the linear benchmark. This hazard function is obtained by imbedding a linear transformation of log-duration within the Logistic Distribution function. The Logistic Distribution function is very smooth, however, and in practice, the Log-Logistic hazard function is quite linear over the observed

range of sample variation. From an empirical perspective, the rate of change in the log-hazard function is essentially constant with both models. One would not ordinarily expect a “linear response” from investors in an environment characterized by explosive price changes. It is this observation that motivates the use of a generalized Weibull model developed by Mudholkar, Srivastava, and Kollia (1996). This model provides much greater flexibility than the conventional Weibull or Log-Logistic hazard models, at the expense of one additional parameter. The generalized Weibull (henceforth Mudholkar) may be better suited to detect rational bubbles in security prices, since it is flexible enough to allow hazard profiles that change at increasing or decreasing rates, and in the extreme, to allow non-monotonic hazard profiles.¹

This study examines the nature of duration dependence in security prices using the Log-Logistic, Weibull, and Mudholkar hazard functions. Runs of positive and negative abnormal monthly returns are constructed for value-weighted portfolios of New York Stock Exchange (NYSE) traded firms. The estimates of the Log-Logistic and Weibull models give conflicting evidence on the direction of duration dependence. The source of this sensitivity to the choice of functional form is revealed by the Mudholkar model, for which the estimated duration elasticity is initially positive, but becomes negative as the length of the run increases. The significant duration dependence found by the Mudholkar model, in runs of both positive and negative abnormal returns, is inconsistent with the rational speculative bubbles model of McQueen and Thorley (1994).

II. Rational Bubbles

Shiller (1989) and Blanchard and Watson (1982) argue that the market price of a stock can deviate from its fundamental value by a rational speculative bubble factor, b_t , provided the bubble factor is expected to grow at the required rate of return, r_t . McQueen and Thorley (1994) offer a

model of a rational bubbles process that allows a bubble to grow, burst, and then restart. In their model, the rational bubble factor must satisfy:

$$\begin{aligned}
 b_{t+1} &= \frac{(1+r_{t+1})b_t}{\pi} - \frac{1-\pi}{\pi}a_0 && \text{with probability } \pi \\
 &= a_0 && \text{with probability } (1-\pi)
 \end{aligned}
 \tag{1}$$

where a_0 is the initial value of the bubble factor and $1-\pi$ is the probability of a crash. With this model, the bubble factor grows by the exact amount needed to compensate investors for the probability that the bubble will crash and return to its initial value.²

As the bubble component grows, it begins to dominate the fundamental component, leading to higher and higher returns prior to the crash. Since a growing bubble component makes negative abnormal returns less likely, McQueen and Thorley (1994) argue that a long run of positive abnormal returns suggests the presence of a price bubble. They contend that if prices contain bubbles, then runs of positive abnormal returns will exhibit negative duration dependence, i.e., an inverse relation exists between the probability of a run ending and the length of the run. In contrast, a rational speculative bubble cannot be negative.³ “Consequently, bubbles generate duration dependence in runs of positive, but not negative, abnormal returns” (McQueen and Thorley, 1994, page 384). As is often the case with theoretical models, the resulting predictions are qualitative in nature, and provide no information about the form of the hazard function. They adopt the log-logistic model for “computational simplicity, not because it conforms to any economic theory or model” (McDonald, McQueen, and Thorley, 1995, footnote 14).

McQueen and Thorley (1994) test for the presence of speculative bubbles under the maintained hypothesis of market efficiency. Absent the rational bubbles factor, stock prices should

behave as a random walk, and there should be no evidence of duration dependence in runs of either positive or negative abnormal returns. The presence of rational bubbles, however, would induce negative duration dependence in runs of positive abnormal returns. A finding of duration dependence in both positive and negative runs suggests inadequacy of the theoretical model; one possible source being a failure of the assumption of market efficiency.

III. Methodology

The Weibull hazard model is perhaps the most frequently used specifications among parametric models of duration. The survivor function of the Weibull model is

$$S(t) = \exp(-\alpha t^{\beta+1}) \quad (2)$$

where $\alpha > 0$, $\beta > -1$, and $t > 0$. The corresponding hazard function is

$$h(t) = \alpha (\beta + 1)t^\beta \quad (3)$$

The fundamental assumption of the Weibull model is a linear relationship between the log of the hazard function and the log of duration.

$$\ln[h(t)] = \ln[\alpha (\beta + 1)] + \beta \ln(t) \quad (4)$$

The parameter β is the duration elasticity of the hazard function.⁴ The Weibull hazard function is monotone in duration, and exhibits positive (negative) duration dependence if β is positive (negative).

The generalized Weibull of Mudholkar, et al. (1996) provides much greater flexibility at the expense of one additional parameter.⁵ This is accomplished by imbedding the conventional Weibull within a larger class of models through the introduction of a second shape parameter. The resulting family “not only contains distributions with unimodal and bathtub hazard shapes, but also allows for a broader class of monotone hazard rates” (Mudholkar, et. al. 1996, pp.1575). In addition to the

conventional Weibull, their model nests a wide variety of popular distributions, including the Burr Type XII (Mudholkar, et. al. 1996, pp.1576).

The survivor function of the Mudholkar model is

$$S(t) = [1 - \lambda \alpha t^{(\beta+1)}]^{-\lambda^{-1}} \quad (5)$$

where $\alpha > 0$, $\beta > -1$, and where the sample space of t is $(0, \infty)$ for $\lambda \leq 0$ and $(0, (\alpha \lambda)^{-\beta^{-1}})$ for $\lambda > 0$. The limit of this survivor function as λ approaches zero is the conventional Weibull survivor function.⁶

The corresponding hazard function is

$$h(t) = \alpha (\beta + 1) t^\beta [S(t)]^{-\lambda} \quad (6)$$

or in log terms

$$\ln[h(t)] = \ln[\alpha (\beta + 1)] + \beta \ln(t) - \lambda \ln[S(t)] \quad (7)$$

Because the log transformation is monotonic, and the survivor function is decreasing in duration and bounded by the unit interval, the hazard function is: monotonically increasing in duration if both β and λ are positive, monotonically decreasing if both β and λ are negative, U-shaped if β is negative and λ is positive, and inverted U-shaped if β is positive and λ is negative. Even when the hazard function of the Mudholkar model is monotone in duration, its log-hazard function is generally non-linear in log-duration. The duration elasticity of the Mudholkar model is

$$\frac{\partial \ln[h(t)]}{\partial \ln(t)} = \beta + \frac{1 - S(t)^\lambda}{S(t)^\lambda} (\beta + 1) \quad (8)$$

Of course, the duration elasticity depends on both the signs and relative magnitudes of β and λ .

The fundamental concept common to all hazard models is the duration elasticity, which, in the case of the Mudholkar model, depends on α , β , and λ . A simple interpretation of λ , in isolation, does not exist. What can be said about λ is that it controls the non-linearity of the hazard profile. When λ is zero, the hazard profile is linear, the duration elasticity is constant, and the hazard

probability of a crash changes at a constant rate. Only non-zero values of λ allow hazard profiles that change at increasing or decreasing rates, or in the extreme, hazard profiles that are non-monotonic. Thus, loosely speaking, λ controls the degree of non-linearity of the hazard profile.

Although securities markets trade continuously, the need to construct a sequence of abnormal returns from variables that are measured at discrete points in time results in a sequence of runs that is measured discretely. The precision with which duration is measured is limited by the unit of time (days, weeks, months, etc.) used to construct the sequence of abnormal returns. A run with an observed length of t_i (an integer) could have been generated by any value of duration within an interval of plus or minus half a time unit about the observed value. Durations are essentially rounded to the nearest unit of time. The log-likelihood function for such a sample is

$$\ln L(\alpha, \beta, \lambda) = \sum_{i=1}^N \left\{ J_i \ln[F(t_i + 0.5) - F(t_i - 0.5)] + (1 - J_i) \ln[1 - F(t_i + 0.5)] \right\} \quad (9)$$

where J_i is a binary variable that indicates whether the observed duration is complete or partial, N is the observed number of expansions (or contractions), and $F(\cdot)$ is the distribution function for observed duration.⁷ Given that the survivor function is the complement of the distribution function, the log-likelihood function may be written as

$$\ln L(\alpha, \beta, \lambda) = \sum_{i=1}^N \left\{ J_i \ln[S(t_i - 0.5) - S(t_i + 0.5)] + (1 - J_i) \ln[S(t_i + 0.5)] \right\} \quad (10)$$

where $S(\cdot)$ is specified as equation (2) for the Weibull model or as equation (5) for the Mudholkar model. McDonald, McQueen, and Thorley (1995) find this to be an effective approach to controlling for discrete observations of duration.⁸

An alternative approach, used by McQueen and Thorley (1994), is to construct a discrete hazard model. If $g(t)$ denotes the discrete density function for duration and $G(t)$ the corresponding distribution function, then the log-likelihood function for a sequence of N runs is

$$\ln L(\alpha, \beta) = \sum_{i=1}^N \{J_i \ln[g(t_i)] + (1 - J_i) \ln[1 - G(t_i)]\} \quad (11)$$

where the discrete density and distribution functions for duration are related as

$$G(t_i) = \sum_{k=1}^{t_i} g(k) \quad (12)$$

The relationship between the hazard function and the density function for completed duration must now be determined. If the hazard probability is defined as the probability that the completed duration equals t , given it is at least t , then for the discrete case we have $h(t) = g(t)/[1-G(t-1)]$. This implies that $g(1)=h(1)$, since $G(0)=0$. This fact, and successive application of the law of conditional probability gives the density for completed duration as

$$g(k) = h(k) \prod_{m=0}^{k-1} [1 - h(m)] \quad (13)$$

for positive integer k , where $h(0)$ is defined as zero.

Choice of a hazard function completes the model. McQueen and Thorley (1994) use the Logistic Distribution function evaluated at a linear transformation of log-duration. Specifically,

$$h(k) = \Psi[\alpha + \beta \ln(k)] = \{1 + \exp[-\alpha - \beta \ln(k)]\}^{-1} \quad (14)$$

where $\Psi(\cdot)$ denotes the Logistic Distribution function. This is a reasonable choice because it makes the hazard probabilities monotone in duration and generates hazard probabilities on the unit interval.

The duration elasticity of the Log-Logistic hazard function is

$$\frac{\partial \ln[h(k)]}{\partial \ln(k)} = \beta \{1 - \Psi[\alpha + \beta \ln(k)]\} = \beta [1 - h(k)] \quad (15)$$

Since the Logistic distribution function is bounded by the unit interval, the Log-Logistic hazard model exhibits positive (negative) duration dependence when β is positive (negative).

Despite the common notation, the parameters α and β in the Log-Logistic and Weibull models are not directly comparable. While the sign of β determines the direction of duration dependence in either model, the duration elasticity of the Log-Logistic hazard is the function

$\beta[1-h(t)]$, whereas the duration elasticity of the Weibull hazard is the constant β . Likewise, the interpretation of α differs between the two models. To illustrate, note that under the restrictions $\alpha=\beta=0$, the value of the Log-Logistic hazard is 0.5, whereas the value of the Weibull hazard is zero.

Of particular interest are conditions under which there is no duration dependence. For the Log-Logistic and Weibull models, there is no duration dependence when β equals zero. For the Mudholkar model, the absence of duration dependence requires that both β and λ equal zero. This joint hypothesis can be tested with a likelihood ratio (LR) test. For ease of comparison, tests of the restriction β equal zero in the Log-Logistic and conventional Weibull model will also be conducted with a LR test.⁹

IV. Data

The samples analyzed in this paper consist of firms traded on the New York Stock Exchange (NYSE) with securities data available in the University of Chicago's Center for Research in Security Prices (CRSP) files. A sequence of monthly abnormal real returns is constructed for value-weighted portfolios of all NYSE stocks from 1927 through 1997. Construction of the series is consistent with the procedures of McQueen and Thorley (1994). Real returns are calculated by subtracting continuously compounded inflation rates from continuously compounded nominal returns.¹⁰ The sequence of real abnormal returns is defined as the residuals from a regression of real returns on its first three lags, the term spread, and the dividend yield.¹¹ Once a time series for abnormal returns is constructed, the corresponding sequences of positive and negative runs is determined, and the length of the run is simply the number of consecutive periods of like sign in abnormal returns. As noted by McQueen and Thorley (1994), any evidence of speculative bubbles must be interpreted as conditional on the specification of the model used to generate the sequence of abnormal returns.

V. Results

We report the estimates obtained for runs of positive and negative abnormal real returns from a value-weighted portfolio in Tables 1 and 2, respectively. In each table, we report Maximum Likelihood estimates for the Log-Logistic, Weibull, and Mudholkar models. We present the point estimates of α , β , and λ , along with likelihood ratio test statistic for the null hypothesis of no duration dependence and for the null hypothesis of a linear log-hazard profile. Figures 1 and 2 plot the relationship between the log-hazard and log-duration for the estimates found in Tables 1 and 2. In these figures, the duration elasticity corresponds to the slope of (a line tangent to) the hazard function.¹²

The estimates of the Log-Logistic model reported in Tables 1 and 2 are consistent with the presence of speculative bubbles. There is evidence of negative duration dependence in runs of positive abnormal returns, but no evidence of duration dependence in runs of negative abnormal returns. For runs of positive abnormal returns, the estimated duration elasticity is negative, and is significantly different than zero at the 5 percent level. This result suggests that the probability of a run of positive abnormal returns ending decreases with the length of the run, as predicted by the McQueen and Thorley (1994) model. For runs of negative abnormal returns, the estimated duration elasticity is again negative, but is not significant at conventional levels. These estimates confirm the results of McQueen and Thorley (1994) with a slightly longer data series.

In contrast, estimates of the Weibull model provide evidence against the presence of speculative bubbles. The Weibull model finds evidence of positive duration dependence in both positive and negative runs of abnormal real returns. In each case, the estimated duration elasticity is positive and significantly different than zero at the 1 percent level. This difference in results is

somewhat curious, given that the log-hazard plots in Figures 1 and 2 reveal that both the Logistic and Weibull hazard are quite linear over the relevant range of variation in the data.¹³

A potential explanation of these differences is provided by estimates of the Mudholkar model. With both positive and negative runs of abnormal real returns, the estimate of β is positive, the estimate of λ is negative, and the time profile of the log-hazard function takes an inverted U-shape. As illustrated by Figures 1 and 2, the direction of duration dependence is not monotone; the duration elasticity is initially positive, but becomes negative as the duration of the run increases.¹⁴ Linearity of the log-hazard profile (obtained under the restriction $\lambda=0$) is rejected at the 1 percent level for both positive and negative runs of abnormal real returns. Likewise, the null hypothesis of no duration dependence (obtained under the joint restriction $\beta=0$ and $\lambda=0$) is rejected at the 1 percent level for both positive and negative runs.

One could speculate that a short initial period of positive duration dependence in runs of positive abnormal returns might be explained by a more general theoretical model of speculative bubbles. Similar evidence of strong duration dependence in runs of negative abnormal returns is more difficult to dismiss, however, and is clearly at odds with the model of rational speculative bubbles. McQueen and Thorley (1994) suggest that significant duration dependence in runs of negative abnormal returns is likely to be the result of a failure of market efficiency.

VI. Conclusion

The results of this paper suggest that the conclusions of McQueen and Thorley (1994) are contingent on the Log-Logistic form of the hazard function. The Log-Logistic and Weibull hazard models, both of which yield log-hazard functions that are quite linear in this application, provide conflicting conclusions regarding the presence of rational speculative bubbles. The Generalized

Weibull model of Mudholkar, et al. (1996), which provides much greater flexibility at the expense of one additional parameter, finds consistent evidence against rational speculative bubbles. The duration elasticity is initially positive, but becomes negative as the duration of the run increases. While the Mudholkar hazard function exhibits negative duration dependence over much of its range, it does so for both positive and negative runs of abnormal real returns. The presence of significant duration dependence in runs of negative abnormal returns is inconsistent with McQueen and Thorley's model of rational speculative bubbles.

It is interesting to note that the parameter estimates and time profile of the log-hazard functions for the Mudholkar model are similar for both positive and negative runs of abnormal returns. This suggests that the evidence of duration dependence may reflect some factor common to the data generation process. One potential explanation is a failure of the assumption of market efficiency. Another possible explanation is the presence of heterogeneity – the effect of unmeasured observation-specific regressors. For negative values of λ , the Mudholkar hazard model is observationally equivalent to a Weibull hazard model with Gamma heterogeneity.¹⁵ With this structure, there is an underlying Weibull hazard model with common duration elasticity β , and observation specific values of α that are Gamma distributed with unit mean and variance $-\lambda$. This interpretation of the Mudholkar estimates suggests that failure to control for heterogeneity leads to significant under-estimation of the duration elasticity.¹⁶

References

Blanchard, O. J. and M. W. Watson. "Bubbles, Rational Expectations, and Financial Markets." In *Crises in the Economic and Financial Structure*, Paul Wachtel (Ed.), Lexington, MA: Lexington Books (1982).

Diba, B. and H. Grossman. "On the Inception of Rational Bubbles." *Quarterly Journal of Economics* 102 (1987), 697-700.

Diba, B. and H. Grossman. "The Theory of Rational Bubbles in Stock Prices." *The Economic Journal* 98 (1988), 746-754.

Fama, E. W. and K. R. French. "Business Conditions and Expected Returns on Stocks and Bonds." *Journal of Financial Economics* 25 (1989), 23-49.

Heckman, J. J. and B. Singer. "Econometric Duration Analysis." *Journal of Econometrics* 24 (1984), 63-132.

Lancaster, T.. "Econometric Methods for the Duration of Unemployment." *Econometrica* 47 (1979), 939-956.

McDonald, J., G. McQueen, and S. Thorley. "Testing for Duration Dependence with Discrete Data." *Working paper*, Brigham Young University (1995).

McQueen, G., and S. Thorley. "Bubbles, Stock Returns, and Duration Dependence." *Journal of Financial and Quantitative Analysis* 29 (1994), 379-401.

Mudholkar, G. S., D. K. Srivastava, and G. D. Kollia. "A Generalization of the Weibull Distribution with Application to the Analysis of Survival Data." *Journal of the American Statistical Association* 91 (1996), 1575-1583.

Prabhakar Murthy, D. N., Min Xie, and Renyan Jiang. *Weibull Models*. Hoboken, NJ: John Wiley and Sons (2003).

Shiller, R. J.. *Market Volatility*. Cambridge, MA: The MIT Press (1989).

Sichel, D. E.. “Business Cycle Duration Dependence: A Parametric Approach.” *Review of Economics and Statistics* 73 (1991), 254-260.

Zuehlke, T. W.. “Business Cycle Duration Dependence Reconsidered.” *Journal of Business and Economic Statistics* 21 (2003), 564-569.

Table 1

The data series is constructed from monthly returns for the period from 1927-1997. Monthly nominal returns are drawn from the CRSP file and inflation rates are drawn from Ibbotson and Associates' (1999) monthly inflation series. Monthly real returns are calculated by subtracting monthly inflation rates from nominal returns. Monthly returns are transformed into series of run lengths on positive abnormal returns, with abnormal returns defined as the residuals from regressions of monthly real returns on the term spread, the dividend yield, and lagged real returns. A run is a series of abnormal returns of the same sign.

The LR statistics test for the absence of duration dependence and the linearity of the log-hazard function. For the Log-Logistic and Weibull models, the absence of duration dependence corresponds to $\beta=0$, and the LR statistic is asymptotically χ^2 with 1 degree of freedom. For the Mudholkar model, the absence of duration dependence corresponds to $\beta=0$ and $\lambda=0$, and the LR statistic is asymptotically χ^2 with 2 degrees of freedom. The log-hazard function of the Mudholkar model is linear under the restriction $\lambda=0$, and the LR statistic is asymptotically χ^2 with 1 degree of freedom

Runs of Positive Abnormal Real Returns			
	Log-Logistic	Weibull	Mudholkar
α	-0.0278	0.3082	0.0667
β	-0.3130	0.3077	12.9340
λ	--	0	-8.3721
LR tests for absence of duration dependence (p-value)			
$H_0: \beta=0$	5.4780 (0.019)	25.3258 (0.001)	
$H_0: \beta=0$ and $\lambda=0$			117.0038 (0.001)
LR test for a linear log-hazard function (p-value)			
$H_0: \lambda=0$			91.6780 (0.001)

Table 2

The data series is constructed from monthly returns for the period from 1927-1997. Monthly nominal returns are drawn from the CRSP file and inflation rates are drawn from Ibbotson and Associates' (1999) monthly inflation series. Monthly real returns are calculated by subtracting monthly inflation rates from nominal returns. Monthly returns are transformed into series of run lengths on negative abnormal returns, with abnormal returns defined as the residuals from regressions of monthly real returns on the term spread, the dividend yield, and lagged real returns. A run is a series of abnormal returns of the same sign.

The LR statistics test for the absence of duration dependence and the linearity of the log-hazard function. For the Log-Logistic and Weibull models, the absence of duration dependence corresponds to $\beta=0$, and the LR statistic is asymptotically χ^2 with 1 degree of freedom. For the Mudholkar model, the absence of duration dependence corresponds to $\beta=0$ and $\lambda=0$, and the LR statistic is asymptotically χ^2 with 2 degrees of freedom. The log-hazard function of the Mudholkar model is linear under the restriction $\lambda=0$, and the LR statistic is asymptotically χ^2 with 1 degree of freedom

Runs of Negative Abnormal Real Returns			
	Log-Logistic	Weibull	Mudholkar
α	0.2410	0.3279	0.1554
β	-0.1371	0.5624	12.9634
λ	--	0	-7.1746
LR tests for absence of duration dependence (p-value)			
$H_0: \beta=0$	0.5578 (0.455)	59.1782 (0.001)	
$H_0: \beta=0$ and $\lambda=0$			132.1408 (0.001)
LR test for a linear log-hazard function (p-value)			
$H_0: \lambda=0$			72.9626 (0.001)

Figure 1: Runs of Positive Abnormal Real Returns

This figure plots the relationship between the log-hazard and log-duration obtained for runs of positive abnormal real returns on a portfolio of value-weighted NYSE-traded securities using the Log-Logistic, Weibull, and Mudholkar models. The duration elasticity corresponds to the slope of the hazard function.

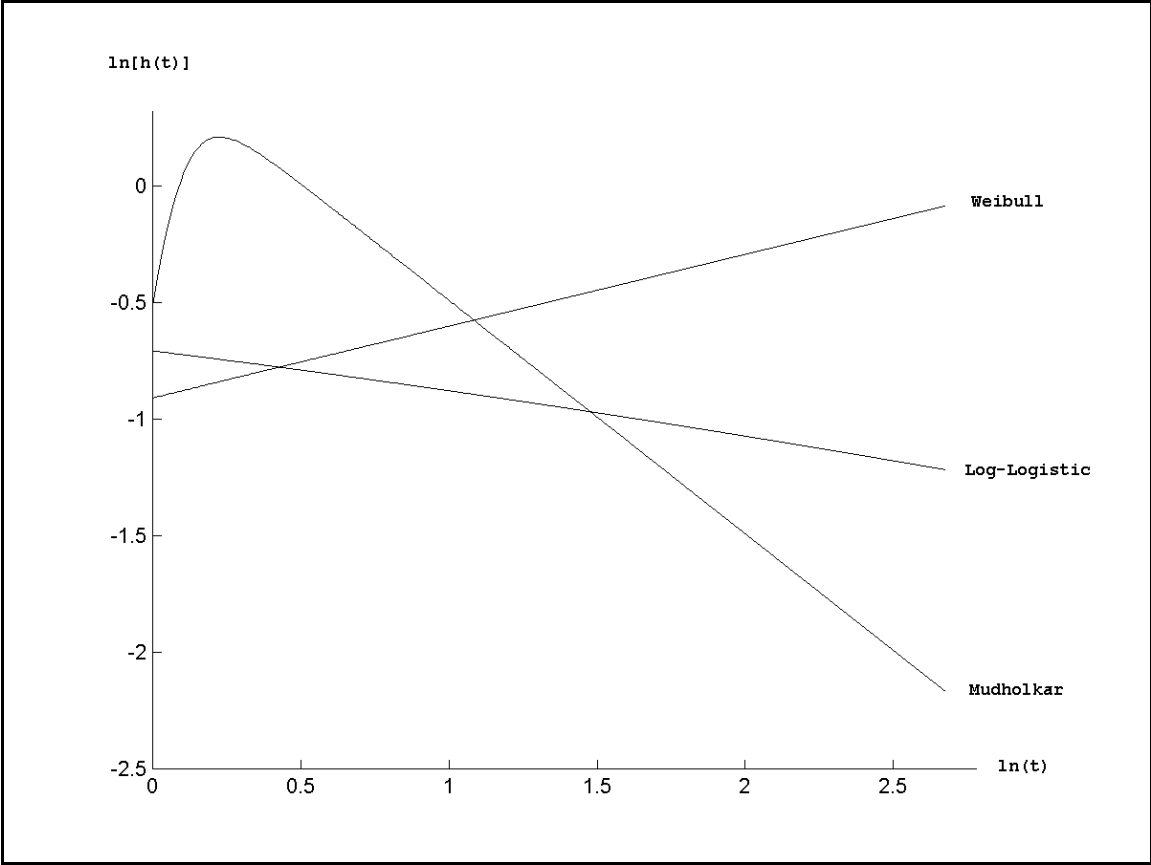
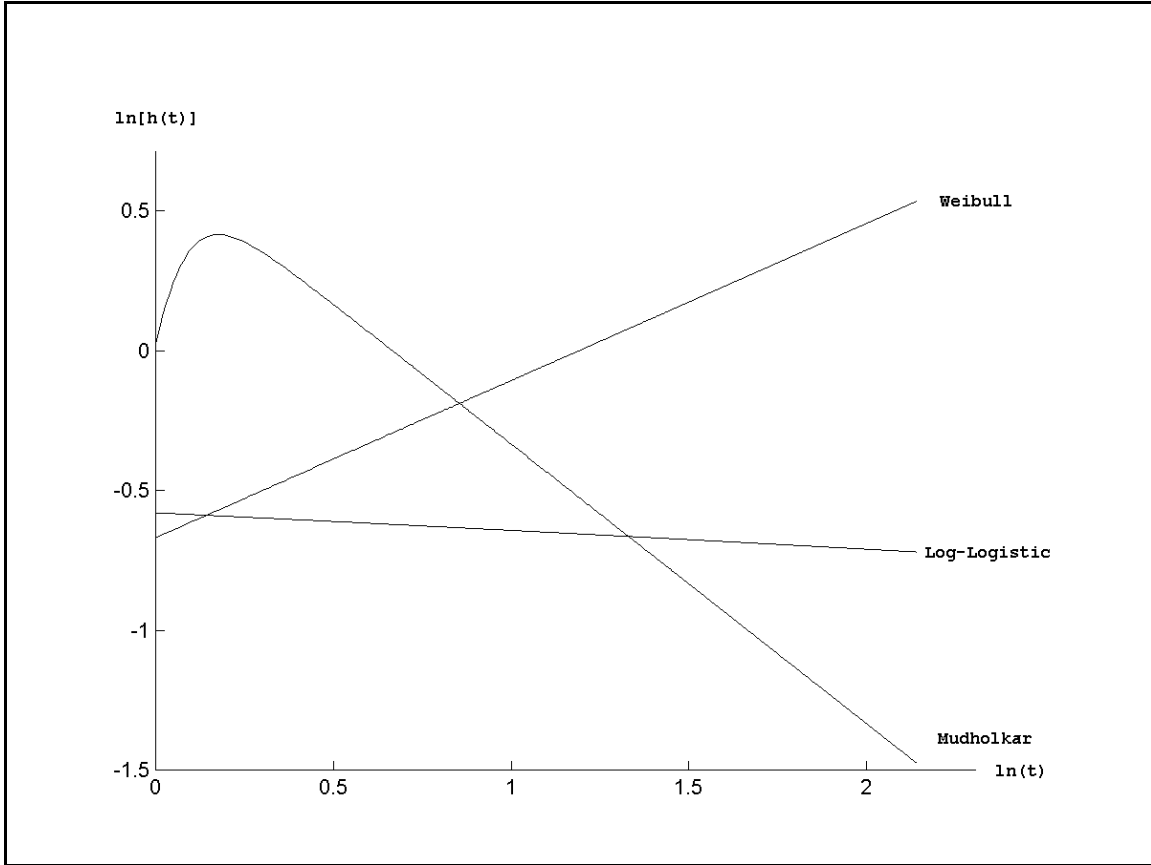


Figure 2: Runs of Negative Abnormal Real Returns

This figure plots the relationship between the log-hazard and log-duration obtained for runs of negative abnormal real returns on a portfolio of value-weighted NYSE-traded securities using the Log-Logistic, Weibull, and Mudholkar models. The duration elasticity corresponds to the slope of the hazard function.



Footnotes

1. Zuehlke (2003) uses the Mudholkar model to update the results of Sichel (1991). In this application, the hazard function for the duration of business cycles is found to increase at an increasing rate.
2. The value of π must be greater than $\frac{1}{2}$ to be consistent with the two traditional characteristics of bubbles: a long run-up in price followed by a crash.
3. Diba and Grossman (1987, 1988) argue that a negative bubble factor is not feasible, since it would grow increasingly negative and eventually lead to negative stock prices.
4. This choice of parameters was established by Lancaster (1979).
5. The term “Generalized Weibull” can be ambiguous. There have been numerous generalizations of the Weibull model, many considerably less flexible than the Mudholkar model, which have borne this label. Prabhakar Murthy, Xie, and Jiang (2003) provide a useful survey of these models.
6. The parameters α , β , and λ , in this paper correspond to $\sigma^{-1/\alpha}$, $(1/\alpha)-1$, and λ in Mudholkar, et al. (1996).
7. While of relative insignificance in larger samples, a partial run will generally occur at the end of the sample period.
8. McDonald, McQueen, and Thorley (1995) provide a Monte Carlo comparison of the performance of the Interval Weibull and discrete Log-Logistic models for the case $\beta=0$ (no duration dependence). Appendices A and B, available on request from the authors, extend these results for $\beta \neq 0$. The power functions of a Likelihood Ratio test for the absence of duration dependence are quite similar for both estimation methods. The conclusion of McDonald, McQueen, and Thorley, that both estimators provide an effective method of controlling for discrete observation of a continuous random variable, appears to be valid even when the data exhibits duration dependence.

9. For the Log-Logistic and Weibull models, the LR test of the restriction $\beta=0$ is asymptotically χ^2 with 1 degree of freedom. For the Mudholkar model, the LR test of the restrictions $\beta=\lambda=0$ is asymptotically χ^2 with 2 degrees of freedom.
10. Continuously compounded inflation rates are constructed from the monthly inflation series available in *Stocks, Bonds, Bills, and Inflation 1999 Yearbook* by Ibbotson Associates. This series is based on the Consumer Price Index without seasonal adjustment.
11. Fama and French (1989) argue that the term spread and dividend yield are useful in predicting time-varying risk premia. The term spread is measured as the difference in yield-to-maturity between the Ibbotson Associates' AAA Corporate Bond Portfolio and the one-month Treasury bill. The dividend yield is that of a value-weighted NYSE portfolio, calculated as the sum of the prior twelve monthly dividends divided by the current price. Both the term spread and the dividend yield are measured at the end of the prior period.
12. While not reported here, the parameter estimates, test statistics, and log-hazard plots for equally-weighted portfolios are quantitatively and qualitatively similar to those obtained with value-weighted portfolios.
13. The log-hazard functions are plotted for values of time between the smallest and largest sample values of duration.
14. Most of the curvature in the Mudholkar log-hazard profile occurs for values of log-duration between 0 and 1. This interval corresponds roughly to values of duration between 1 and 3 months. In terms of the data, this is where the action is, as roughly 85% of the sample of positive runs and 89% of the sample of negative runs have durations less than 3 months.
15. No such interpretation is available for positive values of λ , but as the maximizing values of λ are all negative in this application, this is not a concern.

16. One potential area for future research is to investigate the impact of heterogeneity on the estimated duration elasticity using the semi-parametric methods of Heckman and Singer (1984). This estimator uses a Weibull specification for the hazard function, and the distribution of heterogeneity is estimated non-parametrically.