Duration Dependence in Stock Market Cycles Reconsidered

Yvette S. Harman

Department of Finance
Miami University
Oxford, OH 45056

harmanys@muohio.edu

513-529-5986

and

Thomas W. Zuehlke

Department of Economics
Florida State University
Tallahassee, Florida 32306

tzuehlke@mailer.fsu.edu

850-644-5001

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Abstract

Cochran and Defina (1995) show that evidence of duration dependence implies predictability in stock prices. They estimate a Weibull hazard model using a chronology of stock market cycles and find evidence of duration dependence only for prewar expansions and postwar contractions. This paper updates the postwar sample, and employs a generalized Weibull model that provides much greater flexibility at the expense of one additional parameter. This model finds evidence of duration dependence for all samples, and is statistically superior to the conventional Weibull model for all samples except prewar expansions.

1. Introduction

Cochran and Defina (1995) use a Weibull hazard model to test for duration dependence in U.S. stock market cycles. They observe that stock prices undergo cycles of expansion and contraction, and argue that if cycles in the stock market tend toward a fixed length, their hazard function should exhibit positive duration dependence. That is, the conditional probability a cycle will end is increasing in its duration. If stock cycles are characterized by positive duration dependence, then the duration of the current cycle provides information that is useful in predicting turning points in the stock market. In contrast, Cochran and Defina (1995, pg. 311) note that "if the stock market follows a random walk, deviations from trend are random, market phases do not tend to a fixed length, and duration dependence does not exist." Thus, evidence of duration dependence provides evidence of predictability of stock prices.

The sample employed by Cochran and Defina spans the period January 1885 through July 1992. The estimates obtained by Cochran and Defina provide evidence of positive duration dependence in prewar expansions and postwar contractions, but no evidence of duration dependence in prewar contractions or postwar expansions. This suggests that, in the postwar period, the probability that an expansion will end is unrelated to its current duration, and consequently that there is no statistical evidence to reject the hypothesis that stock prices follow a random walk. At the date of their sample, the stock market was nearing the end of its longest peacetime expansion in history. The expansion that followed lasted even longer, however. This expansion ended in September 2002, which corresponds to a duration of 118 months in length. Given the length of the two most recent stock market expansions, the issue of duration dependence in stock prices is as relevant today as it was a decade earlier.

Cochran and Defina (1995) adopt the methodology of Sichel (1991), who employs a Weibull hazard model to test for duration dependence in the NBER business cycle chronology. Sichel's choice of the Weibull model was motivated by the relatively low power exhibited by nonparametric tests of duration dependence. Even with time series dating back to the mid-1800s, the sample size, which is determined by the number of distinct periods of expansion or contraction, is relatively small. Recent work by Zuehlke (2003b) reconsiders the question of duration dependence in the NBER chronology using the generalized Weibull model of Mudholkar, Srivastava, and Kollia (1996). The fundamental assumption of the Weibull model is a linear relationship between the log of the hazard function and the log of duration. The generalized Weibull model, henceforth the Mudholkar model, provides much greater flexibility than the conventional Weibull model, at the expense of one additional parameter. The Mudholkar model nests the Weibull model, and allows nonlinear and nonmonotonic profiles for the log-hazard function, while providing a "proper" hazard function for all parameter values.[1]

It should be noted that the hazard function is a reduced form equation. In general, estimates of reduced form parameters tend to be robust since they do not impose the restrictions of any particular structural model. The corresponding disadvantage, is that absent their structural restrictions, one cannot distinguish among competing structural models. In the current application, there are any number of structural models of stock price predictability consistent with the existence of positive duration dependence. Cochran and Defina (1995, p.309) consider the possibility of "temporary 'fads' in the pricing of securities or ... time-varying required returns." Campbell (2000) provides a survey of the recent literature on models of stock price predictability.

This paper extends the work of Cochran and Defina in two directions. First, the Mudholkar model is estimated using their original stock price chronology. Estimates of the Weibull model are included for purpose of comparison. Second, the sample period is extended through September 2002. This will allow us to determine whether the results of Cochran and Defina are sensitive to the log-linear specification inherent in the Weibull model or the new sample information reflected in the most recent expansion.

2. Data

Cochran and Defina (1995) provide a series of reference dates for stock market cycles over the period January 1885 through July 1992. Their chronology is constructed from information provided by Moore and Cullity (1988) and Cohen, Zinbarg, and Zeikel (1987). This paper employs the original series provided by Cochran and Defina, with the following corrections. The expansion covering June 1888 through May 1890 is listed as 11 months in Cochran and Defina. The duration of this expansion has been corrected to 23 months, after confirming the reference dates in Moore and Cullity. Likewise, the durations of the expansion of March 1978 through December 1980, and the subsequent contraction of December 1980 through July 1982, were revised from 32 and 20 months respectively, to 33 and 19 months, again following confirmation of the reference dates in Moore and Cullity.

We also extend the chronology of stock market reference dates through September 2002. In extending this series, we follow the methodology of Moore and Cullity. This involves comparing peaks and troughs in the NBER business cycle chronology with their counterparts in the S&P 500 index. This process suggests a minor revision to the chronology provided by Cochran and Defina.

There is a contraction in the NBER business cycle chronology from July 1990 through March 1991. There is a corresponding contraction in the S&P 500 index from May 1990 through October 1990. The S&P 500 index fell by about 14 percent over this 5 month period. This contraction occurs near the end of the 120 month incomplete expansion that concludes their sample period. It is likely that Cochran and Defina could not confirm the correspondence of this contraction in the S&P 500 with its counterpart in the NBER chronology, since decisions of the NBER Business Cycle Dating Committee are made *ex post*, and are reported with a substantial delay. The next peak in the NBER business cycle chronology occurs on March 2001, with corresponding peak in the S&P 500 index on August 2000. These reference dates are used to extend the stock market chronology through September 2002. Both the original series of Cochran and Defina (with corrections noted in the previous paragraph) and the extended series are presented in Appendix A.

3. Methodology

A hazard model may also be specified in terms of its survivor function. The survivor function of the Weibull model is

$$S(t) = \exp(-\alpha t^{\beta+1}) \tag{1}$$

where $\alpha > 0$, $\beta > -1$, and $t > 0$. The corresponding hazard function is

$$h(t) = \alpha(\beta + 1)t^{\beta} \tag{2}$$

or in log terms

$$\ln[h(t)] = \ln[\alpha(\beta + 1)] + \beta \ln(t) \tag{3}$$

This is the fundamental assumption of the Weibull specification; a linear relationship between the log of the hazard function and the log of duration. The parameter β is the duration elasticity of the

Weibull hazard model. The Weibull hazard function is monotone in duration, and exhibits positive (negative) duration dependence when β is positive (negative).[2]

Mudholkar, Srivastava, and Kollia (1996) present a generalized Weibull model that provides much greater flexibility at the expense of one additional parameter. Their generalized model allows hazard functions that are monotically increasing, monotonically decreasing, U-shaped, and inverted U-shaped. The survivor function of the Mudholkar model is

$$S(t) = [1 - \lambda \alpha t^{(\beta+1)}]^{\lambda^{-1}} \tag{4}$$

where $\alpha>0$, $\beta>-1$, and where the sample space of t is $(0,\infty)$ for $\lambda\leq 0$ and $(0, (\alpha\lambda)^{-(1/\beta)})$for $\lambda>0$. The survivor function of the Mudholkar model converges to that of the Weibull model as λ approaches zero. The hazard function of the Mudholkar model is

$$h(t) = \alpha(\beta + 1)t^{\beta}[S(t)]^{-\lambda} \tag{5}$$

or in log terms

$$\ln[h(t)] = \ln[\alpha(\beta + 1)] + \beta \ln(t) - \lambda \ln[S(t)] \tag{6}$$

Since the log transformation is monotonic, and the survivor function is decreasing in duration and bounded by the unit interval, the hazard function is: monotonically increasing in duration if both β and λ are positive, monotonically decreasing if both β and λ are negative, U-shaped if β is negative and λ is positive, and inverted U-shaped if β is positive and λ is negative.[3]

Even when the hazard function of the Mudholkar model is monotone in duration, its log-hazard function is generally nonlinear in log-duration. The duration elasticity of the Mudholkar model is

$$\frac{\partial \ln[h(t)]}{\partial \ln(t)} = \beta + \frac{1 - S(t)^{\lambda}}{S(t)^{\lambda}}(\beta + 1) \tag{7}$$

In general, the duration elasticity depends on the signs and relative magnitudes of both β and λ.

In the Weibull model, a significant estimate of β provides statistical evidence of duration dependence. In the Mudholkar model, a significant estimate of <u>either</u> β or λ provides statistical evidence of duration dependence. A significant estimate of λ also provides evidence of the statistical superiority of the Mudholkar model relative to the Weibull model, because a nonlinear time profile for the log-hazard function can only occur when λ is nonzero.

Cochran and Defina (1995) model the sequences of expansions and contractions separately. Their log-likelihood function corrects for two types of censoring. First, when a cycle is still in progress at the ending date of the sample, its complete duration is censored. All that is known is that the complete duration of the cycle is no less than its observed value. The log-likelihood function for a sequence of expansions (contractions) with observed durations, t_i, is

$$\ln L(\alpha, \beta, \lambda) = \sum_{i=1}^{N} \left\{ J_i \ln[f(t_i)] + (1 - J_i) \ln[1 - F(t_i)] \right\} \tag{8}$$

where J_i is a binary variable that indicates whether the cycle is complete, N is the number of expansions (contractions) in the sequence, and where f( ) and F( ) are, respectively, the density function and distribution function for duration. Second, expansions (contractions) of relatively short duration may be censored. Cochran and Defina (1995, pg. 311) argue that "a transition to a new phase is observed only if that state persists for a minimum period of time." This second form of censoring is controlled for by using the density and distribution functions for duration, conditional on duration exceeding some minimum, denoted δ. This censoring threshold is estimated as one period less than the minimum value of observed duration for the sample period.[4] The resulting log-likelihood function is

$$\ln L(\alpha, \beta, \lambda) = \sum_{i=1}^{N} \left\{ J_i \ln\left[\frac{f(t_i)}{1 - F(\delta)}\right] + (1 - J_i) \ln\left[\frac{1 - F(t_i)}{1 - F(\delta)}\right] \right\} \tag{9}$$

Given that the survivor function is the complement of the distribution function, and the density function is the product of the hazard function and survivor function, this log-likelihood function may be written as

$$\ln L(\alpha,\beta,\lambda) = \sum_{i=1}^{N} \left\{ J_i \ln[h(t_i)] + \ln[S(t_i)] - \ln[S(\delta)] \right\} \tag{10}$$

where S( ) and h( ) are specified as equations (1) and (2) for the Weibull model, or as equations (4) and (5) for the Mudholkar model.

The parameters are estimated by the method of Maximum Likelihood (ML). The score equations for α, β, and λ are a set of simultaneous nonlinear implicit functions, and must be solved numerically. Typically, asymptotic standard errors are reported with nonlinear ML estimates. This paper follows Sichel (1991), however, and reports bootstrapped standard errors, as well as p-values for bootstrapped t-statistics, because of the relatively small sample sizes found in the stock market chronology.[5]

4. Results

Tables 1 and 2 present the estimates obtained for stock market expansions and contractions, respectively. Maximum Likelihood estimates of the parameters of the Weibull and Mudholkar models are presented, along with standard errors based on 500 bootstrapped samples, and the p-value of the corresponding t-statistic. Following Sichel (1991), these p-values are calculated for a one-tailed test using the bootstrapped standard errors. Figures 1A and 1B plot the relationship between the hazard function and duration for prewar and postwar expansions, while Figures 2A and 2B do likewise for prewar and postwar contractions. Because of differences in the range of sample variation, the horizontal axis of each figure is scaled to match the observed range of sample variation. The gradations on the horizontal axis correspond to the observed values of duration within the sample.

The first panel of Table 1 provides estimates for expansions in the prewar sample period (January 1885 through April 1942). The duration elasticity of the Weibull model, β, is estimated to be 0.702, which is significantly positive at the 10% level. The estimate of β in the Mudholkar model is 2.094, which is significantly positive at the 5% level. The corresponding estimate of λ is 0.037, which is not significant at conventional levels. Hence, while both models find evidence of positive duration dependence, the Mudholkar model provides, at best, a modest improvement in fit, as the hypothesis λ=0 cannot be rejected.

The second panel of Table 1 provides estimates for expansions in Cochran and Defina's postwar sample period (February 1948 through July 1992). The estimated duration elasticity for the Weibull model is 0.046, which is not statistically significant. The estimates of β and λ in the Mudholkar model are also insignificant. The Mudholkar model does provide weak evidence of duration dependence, however. The estimate of λ, while insignificant at conventional levels, is marginally significant at an α level of roughly 12%.

The third panel of Table 1 provides estimates for expansions in the updated postwar sample period (February 1948 through September 2002). The estimated duration elasticity for the Weibull model rises to approximately 0.231, but remains insignificant at conventional levels. The estimate of β in the Mudholkar model also remains insignificant. The estimate of λ, however, is now significantly positive at the 10% level. As noted in the previous section, a significant estimate of λ not only provides evidence of duration dependence, but of the statistical superiority of the Mudholkar model relative to the Weibull model as well.

The nature of the improvement provided by the Mudholkar model is evident in the hazard plots, which are qualitatively similar for the prewar sample (Figure 1A) and the postwar samples (Figure 1B). The duration elasticities of the Mudholkar model are positive and increasing. While quite similar to those of the Weibull model for expansions of relatively short duration, the duration elasticity of the Mudholkar model increases rapidly for expansions of longer duration. The improvement in fit with the Mudholkar model occurs in the tail of the distribution.

A comparison of the results for the two postwar samples suggests that the greater flexibility of the Mudholkar model and the new sample information in the latest expansion are both important to the finding of positive duration dependence for expansions in the postwar period. Use of the updated sample alone does not lead to a significant duration elasticity in the Weibull model. Use of the Mudholkar model with the postwar sample of Cochran and Defina does not result in a finding of significant duration dependence. Only when the Mudholkar model is used in conjunction with the updated sample do we get significant duration dependence. These results suggest the not too surprising conclusion, that given the relatively small sample of 11-12 observations in the postwar period, use of a more flexible model or the addition of new observations improves the precision of the estimates. The insignificant duration elasticities for the postwar period found in Cochran and Defina (1995) may be, at least in part, a small sample problem.

The first panel of Table 2 provides estimates for the prewar sample of stock market contractions. The estimated duration elasticity of the Weibull model is 0.296, which is not significantly positive at conventional levels. For the Mudholkar model, the estimate of β is not significant, but the estimate of λ is significantly positive at the 5% level. The duration elasticity is positive and increasing, and the Mudholkar model again represents a significant improvement over the Weibull model. For the prewar sample of contractions, the Mudholkar model is capable of detecting duration dependence when the Weibull is not.

The second panel of Table 2 reports the estimates obtained with Cochran and Defina's postwar sample of contractions. Strong evidence of positive duration dependence is found. The estimated duration elasticity of the Weibull model is 2.157 and is significantly positive at the 5% level. As with the prewar sample of contractions, the estimate of β in the Mudholkar model is not significant, but the estimate of λ is significantly positive at the 5% level. The Mudholkar model again represents a significant improvement over the Weibull model.

The third panel of Table 2 presents the estimates obtained with the updated postwar sample. The point estimates are qualitatively similar to those obtained with Cochran and Defina's postwar sample of contractions. The estimated duration elasticity of the Weibull model falls slightly to 1.211, and is now significantly positive at the 10% level. Likewise, the estimate of β in the Mudholkar is insignificant, while the estimate of λ is significantly positive at the 10% level. Figures 2A and 2B present hazard plots for prewar and postwar contractions. The plots are qualitatively similar to those found with expansions. The duration elasticities of the Weibull and Mudholkar models are similar for contractions of relatively short duration, with the duration elasticity of the Mudholkar model increasing as the contraction persists.

5. Conclusion

The Weibull estimates presented in the first two panels of Tables 1 and 2 replicate the results of Cochran and Defina (1995). The estimated duration elasticities of Cochran and Defina, obtained by subtracting one from their estimate of β, are quantitatively similar to those reported here.[6] Most of the differences in the estimated duration elasticities are the result of the corrections we made to the Cochran and Defina chronology. The estimated duration elasticities for prewar and postwar contractions are almost identical. This is because only minor corrections were made to the data series for contractions. In the prewar period, there were no corrections, and in the postwar period, the only correction was to reduce the duration of the December 1980 through July 1982 contraction from 20 to 19 months. Our estimated duration elasticity for prewar expansions is slightly larger than that of Cochran and Defina, no doubt the result of correcting the length of the June 1888 through May 1890 expansion from 11 to 23 months. The only correction made to the postwar sample of expansions was to increase the duration of the March 1978 through December 1980 expansion from 32 to 33 months. Curiously, we could not replicate the estimates reported by Cochran and Defina for this sample, even when we used their original chronology. Although statistically insignificant, the duration elasticities they report for this sample correspond to negative duration dependence. The point estimates we obtain using their chronology are also statistically insignificant, but correspond to positive duration dependence. None of this affected the qualitative conclusions. Significant evidence of duration dependence is only found for prewar expansions and postwar contractions. Use of the updated postwar samples does not change the qualitative nature of the results obtained with the Weibull model.

The estimates of the Mudholkar model obtained with the prewar and updated postwar samples provide a different set of conclusions. With the prewar samples, evidence of positive duration dependence is found for both expansions and contractions. Furthermore, the estimate of the parameter λ, which distinguishes the two models and provides a statistical test of the superiority of the Mudholkar model, is found to be statistically positive for prewar contractions. When applied to the updated postwar samples, the Mudholkar model again finds evidence of positive duration dependence for both expansions and contractions, and the estimate of λ is found to be significantly positive for both expansions and contractions. In this application, the statistical advantage of the Mudholkar model relative to the Weibull model, is in its ability to allow positive and increasing duration elasticities. By applying this more flexible model to the updated stock market chronologies, evidence of duration dependence, and hence stock price predictability, is found for all sample periods.

Appendix A: Reference Dates and Durations

Trough	Peak	Prewar ('85-'42) Contraction	Prewar ('85-'42) Expansion	Postwar ('48-'92) Contraction	Postwar ('48-'92) Expansion	Postwar ('48-'02) Contraction	Postwar ('48-'02) Expansion
Jan-1885	May-1887		28				
Jun-1888	May-1890	13	23				
Dec-1890	Aug-1892	7	20				
Mar-1895	Sep-1895	31	6				
Aug-1896	Apr-1899	11	32				
Sep-1900	Sep-1902	17	24				
Oct-1903	Sep-1906	13	35				
Nov-1907	Dec-1909	14	25				
Jul-1910	Sep-1912	7	26				
Dec-1914	Nov-1916	27	23				
Dec-1917	Jul-1919	13	19				
Aug-1921	Mar-1923	25	19				
Oct-1923	Sep-1929	7	71				
Jun-1932	Feb-1934	33	20				
Mar-1935	Feb-1937	13	23				
Apr-1938	Oct-1938	14	6				
Apr-1942		42					
Feb-1948	Jun-1948			21	4	21	4
Jun-1949	Jan-1953			12	43	12	43
Oct-1953	Jul-1956			9	33	9	33
Dec-1957	Jul-1959			17	19	17	19
Oct-1960	Dec-1961			15	14	15	14
Jun-1962	Jan-1966			6	43	6	43
Oct-1966	Dec-1968			9	26	9	26
Jun-1970	Jan-1973			18	31	18	31
Dec-1974	Sep-1976			23	21	23	21
Mar-1978	Dec-1980			18	33	18	33
Jul-1982	May-1990			19	120 +	19	94
Oct-1990	Aug-2000					5	118
Sep-2002						25 ++	
Mean Duration (pre)		17.94	25.00				
Mean Duration (post)				15.18	35.18	15.15	39.92

Note: The duration of each expansion or contraction is listed adjacent to its ending date. The + indicates the effective duration of the incomplete expansion (as of Jul-1992) employed in Cochran and Defina (1995). The ++ indicates the effective duration of the incomplete contraction (as of Sep -2002).

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Table 1: Parameter Estimates for Stock Market Expansions						
Prewar Sample (January 1885 - April 1942)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.00376	0.00617	0.27613	0.00005	0.00028	0.42823
β	0.70173	0.47108	0.07925	2.09353	1.16835	0.04822
λ	0	--	--	0.03680	0.04110	0.19343
Cochran and Defina's Postwar Sample (February 1948 - July 1992)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.02341	0.04160	0.29367	0.00048	0.00196	0.40665
β	0.04607	0.44880	0.46025	1.24512	0.99999	0.12416
λ	0	--	--	0.04480	0.06455	0.25367
Updated Postwar Sample (February 1948 - September 2002)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.01028	0.01332	0.22912	0.01723	0.03415	0.31302
β	0.23139	0.28796	0.22017	0.08302	0.50284	0.43626
λ	0	--	--	0.33100	0.23544	0.09667

Note: SE denotes bootstrap standard errors. P-value denotes the marginal significance level of a one-sided test using the bootstrap standard errors.

Table 2: Parameter Estimates for Stock Market Contractions						
Prewar Sample (January 1885 - April 1942)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.02905	0.04366	0.25833	0.13743	0.32228	0.33839
β	0.29638	0.38538	0.22732	- 0.27673	0.65038	0.33872
λ	0	--	--	0.48740	0.19489	0.01327
Cochran and Defina's Postwar Sample (February 1948 - July 1992)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.00014	0.00051	0.39585	0.00169	0.00771	0.41611
β	2.15654	1.04015	0.03400	1.19974	1.28060	0.18812
λ	0	--	--	0.59880	0.23433	0.01694
Updated Postwar Sample (February 1948 - September 2002)						
	Weibull			Mudholkar		
	MLE	SE	P-value	MLE	SE	P-value
α	0.00194	0.00570	0.36995	0.00841	0.02837	0.38646
β	1.21134	0.82017	0.08387	0.61158	1.01944	0.28095
λ	0	--	--	0.66410	0.43131	0.07732

Note: SE denotes bootstrap standard errors. P-value denotes the marginal significance level of a one-sided test using the bootstrap standard errors.

Figure 1A: Hazard Plots for Pre-War Stock Market Expansions

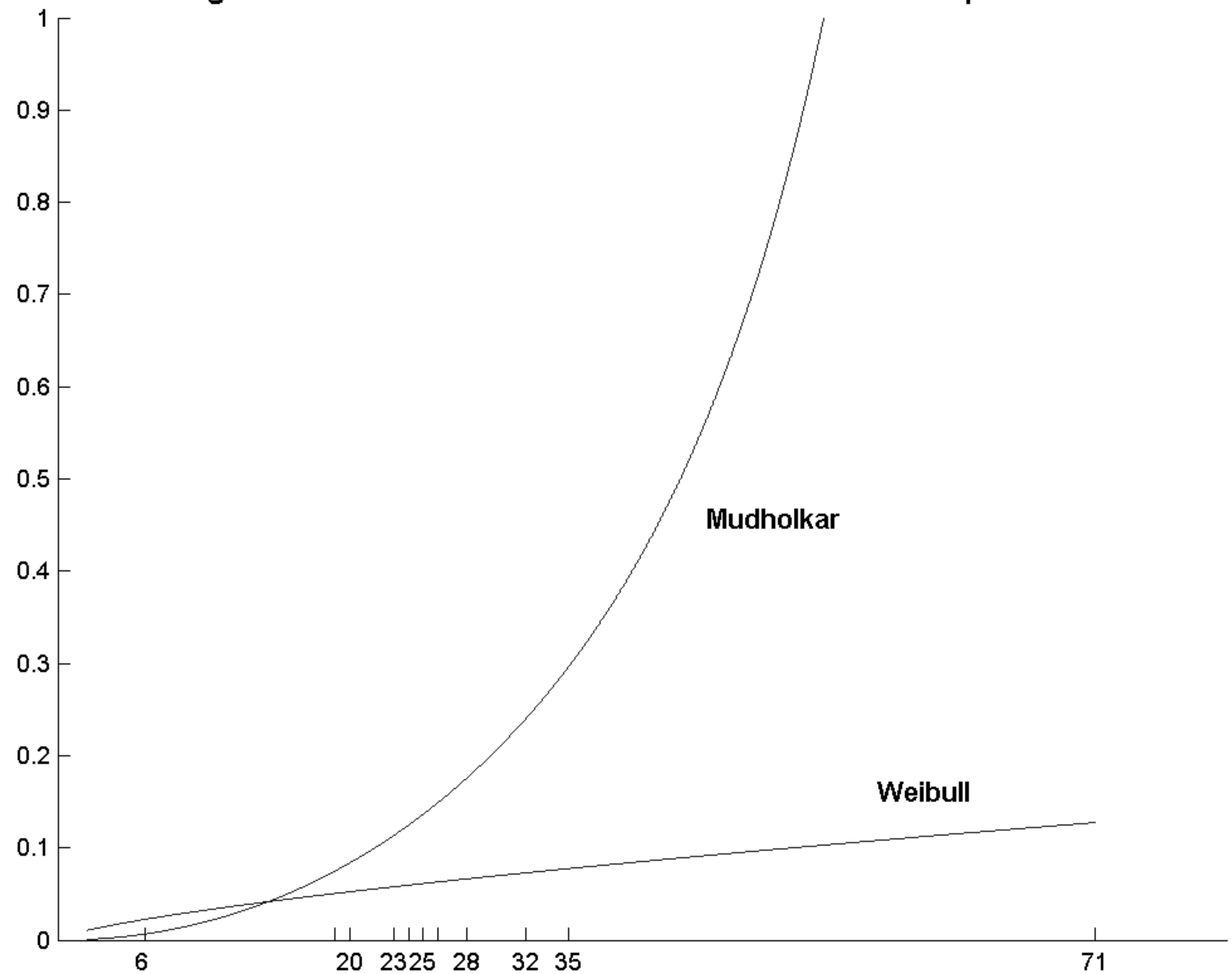


Figure 1B: Hazard Plots for Post-War Stock Market Expansions

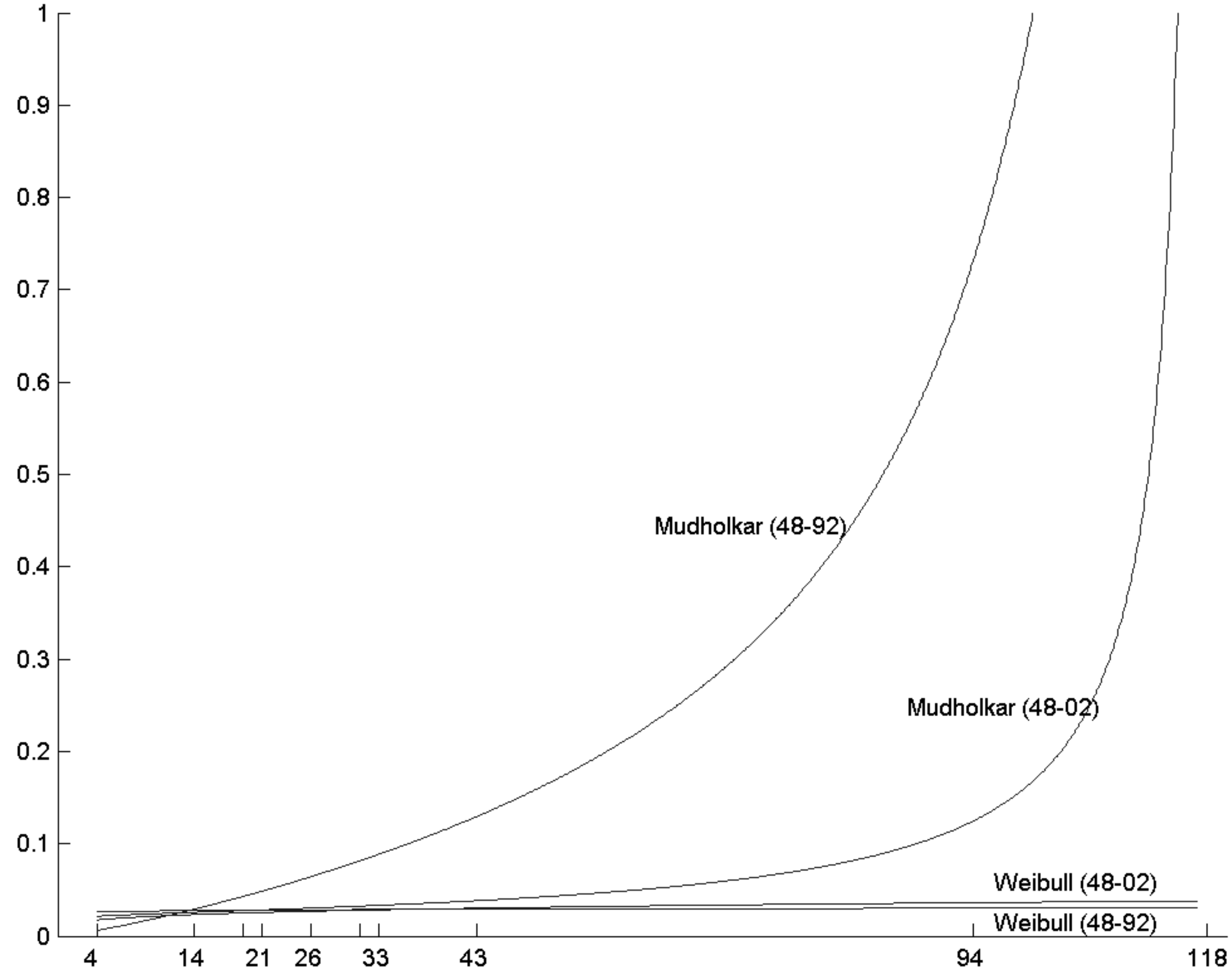


Figure 2A: Hazard Plots for Pre-War Stock Market Contractions

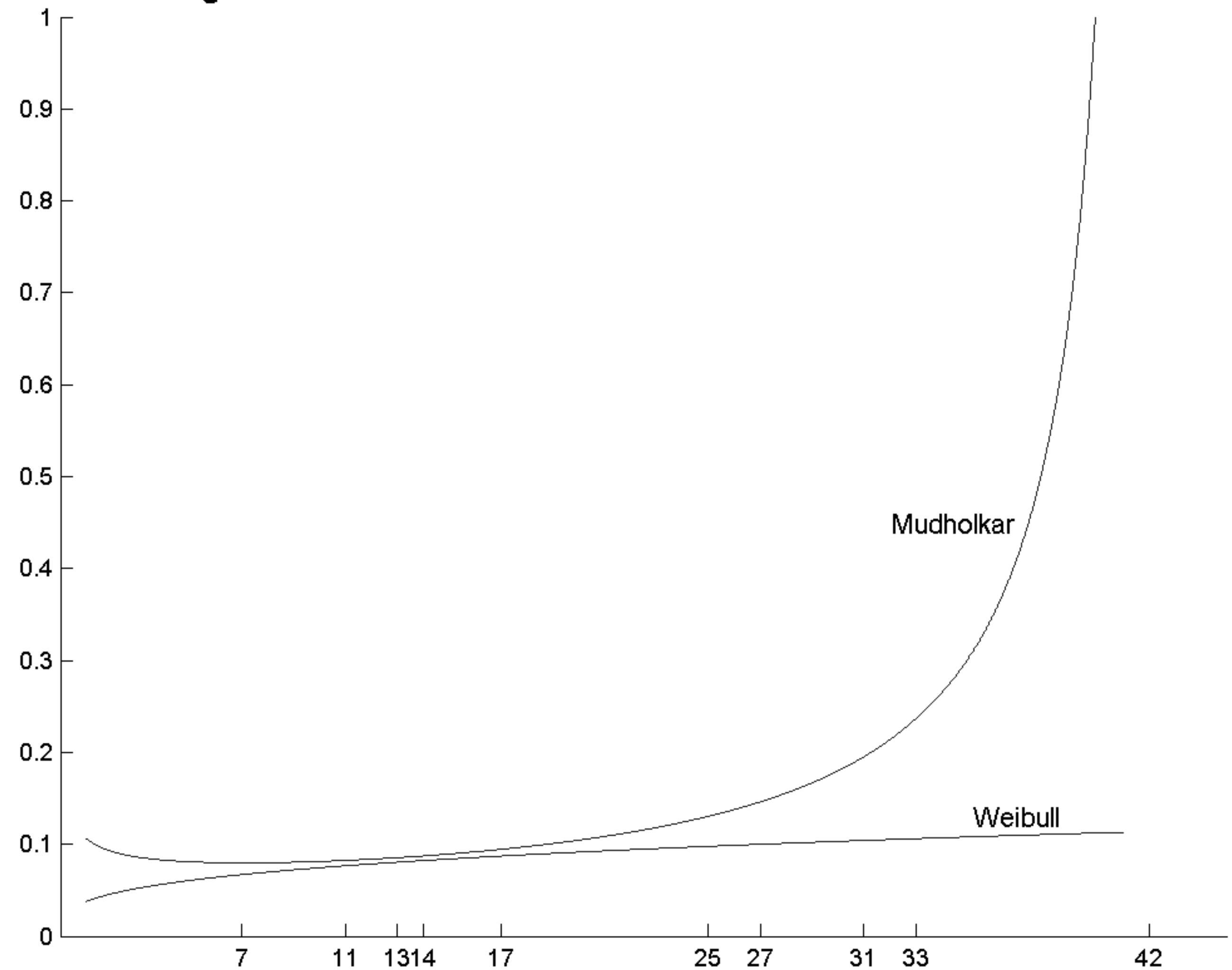
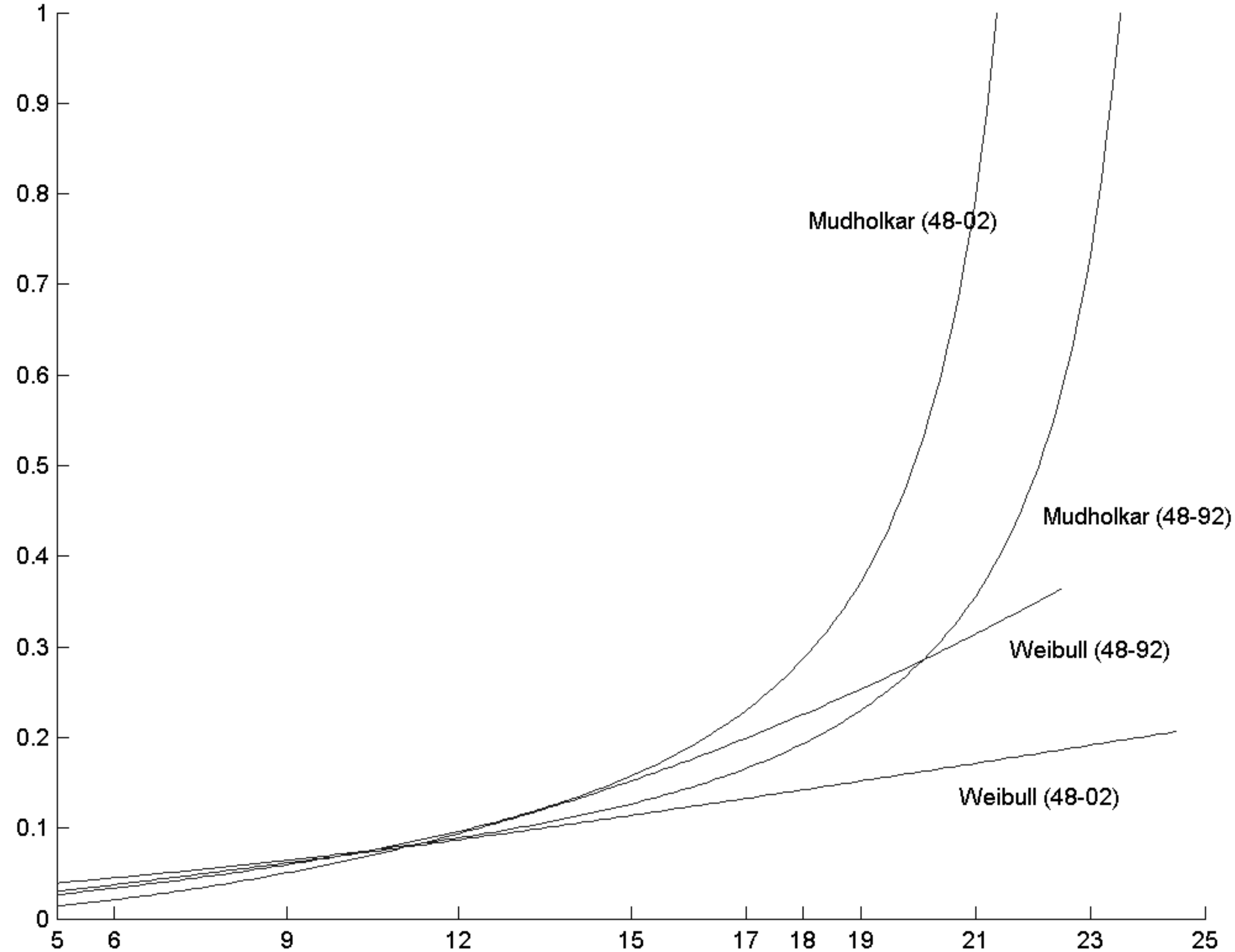


Figure 2B: Hazard Plots for Post-War Stock Market Contractions



Footnotes

1. In order to gain increased flexibility in the time profile of the hazard function, some researchers have utilized local approximations that result in hazard functions that do not satisfy the laws of probability. Specifically, a hazard function must be non-negative with a survivor function that approaches zero as duration approaches infinity. See, for example, Diebold, Rudebusch, and Sichel (1993).

2. The parameters α and β in this paper correspond to α and β-1 in Cochran and Defina (1995).

3. The parameters α, β, and λ, in this paper correspond to $\sigma^{-(1/\alpha)}$, $(1/\alpha)$-1, and λ in Mudholkar, Srivastava, and Kollia (1996).

4. This is the ML estimate of δ. See Zuehlke (2003a).

5. Efron and Tibshirani (1993) give a detailed discussion of the bootstrap method, and Bickel and Freedman (1981) consider the asymptotic validity of bootstrapped t-statistics.

6. Cochran and Defina conduct a sensitivity analysis over values of δ, which they denote d_0. Their Table 3 reports estimates of β for all integer values of δ in the interval 0 to t_1, where t_1 denotes the smallest value of duration in the sample. Cochran and Defina (1995, pg. 312) conclude that "the estimates of β are relatively insensitive to the choice of d_0." In this paper, we only report estimates corresponding to the ML estimator of δ, which is t_1-1. The estimated values of δ in this paper are: 5 for prewar expansions, 6 for prewar contractions, 3 for postwar expansions, and 5 for postwar contractions.