Testing for Interaction in Binary Logit and Probit Models: Is a Product Term Essential?*

ABSTRACT

Political scientists presenting binary dependent variable (BDV) models often offer hypotheses that independent variables interact in their influence on the probability that an event *Y* occurs, Pr(*Y*). A consensus appears to have evolved on how to test such hypotheses: (*i*) estimate a logit or probit model including product terms to specify the interaction, (*ii*) test the hypothesis by determining whether the coefficients for these terms are statistically significant, and (*iii*) if they are, describe the nature of the interaction by estimating how the marginal effect of one independent variable on Pr(*Y*) varies with the value of the other independent variables. We contend that in the BDV context, statistically significant product term coefficients are neither necessary nor sufficient for concluding that there is substantively meaningful interaction among variables in their influence on Pr(*Y*). Even when no product terms are included in a logit or probit model, if the marginal effect of one variable on Pr(*Y*) is related to another independent variable then substantively meaningful interaction is present, and describing such interaction is essential to an accurate portrayal of the data generating process at work. We propose a strategy for studying interaction in the BDV context that is consistent with the recent emphasis in the discipline on casting hypotheses in terms of effects on the probability of an event's occurrence and reporting estimated marginal effects on this probability.

*A replication data set for this study is available at http://www....edu.

Introduction

Many phenomena important to political scientists are binary outcomes: an event occurs, or it does not. In 2005, *American Journal of Political Science (AJPS)*, *American Political Science Review (APSR)*, and *Journal of Politics (JOP)* published 49 quantitative articles analyzing binary dependent variables (BDVs), representing 39% of all the empirical papers in these journals. In the vast majority of this work, logit or probit was the estimator of choice.

A consensus has evolved about how to frame and interpret BDV models. Recent work has sensitized political scientists to the need to extract substantively meaningful information from BDV models and convey interpretations of the effects of independent variables that anyone can understand (King, Tomz and Wittenberg 2000). Previously, the norm among political scientists reporting logit or probit results for BDV models was to restrict attention to the sign and statistical significance of the maximum likelihood estimates (MLEs) for coefficients. Now, researchers analyzing a BDV are urged to frame their hypotheses in terms of the effects of variables on the probability that an event will occur, estimate a logit or probit model, and then use the MLEs for coefficients to compute estimates of the effects of variables on this probability (King 1989, section 5.2; Long 1997, section 3.7).[1]

Political scientists presenting BDV models frequently offer hypotheses that independent variables *interact*. Two independent variables are said to interact in influencing a dependent variable when the marginal (or instantaneous) effect of one independent variable varies with the value of the other. Indeed, of the 49 papers presenting BDV models in the 2005 issues of *APSR*, *AJPS* and *JOP*, twelve—nearly one-quarter—test hypotheses that one or more variables interact.

[1] Fully 98% of the BDV models appearing in the three general political science journals in 2005 employed coefficient MLEs to generate estimates of the effects of variables on the probability that the BDV assumes one of its values.

A consensus also seems to have evolved on how to empirically test a theory that two variables, X_1 and X_2, interact in influencing a BDV:

1. Let the dependent variable be the probability that some event, Y, will occur—to be denoted $\Pr(Y)$.

2. Develop a hypothesis about how the effect of X_1 on $\Pr(Y)$ should vary with the value of X_2, when other independent variables ($X_3, X_4,..., X_k$) are fixed at specified values (typically, their mean).

3. Estimate a model $\Pr(Y) = G\left(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_p X_1 X_2 + \sum_{i=3,...,k} \beta_i X_i\right)$, where $G(\,)$ is a logit or probit link function.

4. Determine whether the coefficient for the product term (β_p) is statistically significant and in the predicted direction.

    a. If β_p is not significant, reject the hypothesis of interaction.

    b. Otherwise, use the coefficient MLEs to compute estimates of the effect of X_1 on $\Pr(Y)$ at different values of X_2 when $X_3, X_4,..., X_k$ are fixed at the specified values, and determine how this effect varies with X_2. This can be accomplished by simulating first differences (King, Tomz, and Wittenberg 2000) or by estimating the marginal effects and their variance and producing a plot (Brambor, Clark and Golder 2006).

Indeed, eleven of the twelve articles analyzing a BDV model and positing interaction in the 2005 issues of *AJPS*, *APSR* or *JOP* estimate a model including a product term and use the coefficient MLEs to estimate how the effect of one independent variable on $\Pr(Y)$ varies with the value of another.

The reliance on this practice is likely a response to two recommendations in the political methodology literature for applied researchers examining a BDV. First, scholars ought to focus on the presentation and interpretation of substantively relevant quantities—typically predicted probabilities, $\Pr(Y)$, in the BDV context. Estimated logit or probit coefficients are generally "difficult to interpret and only indirectly related to the substantive issues that motivated the research" (King, Tomz, and Wittenberg 2000, 348; see also Huang and Shields 2000). Indeed, "the [logit or probit] analyst is not concerned with model parameters per se; he or she is primarily interested in the marginal effect of X_1 on Pr(Y) for substantively meaningful values of the conditioning variable" X_2 (Brambor, Clark, and Golder 2006, 74).

A second recommendation advanced is that a statistically significant product term coefficient, β_p, is a necessary condition for concluding that meaningful interaction takes place. The functional form of logit and probit guarantees that the marginal effect of each independent variable on Pr(Y) is conditional on the value of every independent variable in the model. However, any finding of interaction without a product term is an "artifact of the methodology," and thus substantively meaningless (Nagler 1991, 1393). Nagler (1994, 250) writes that "a simple test for the existence of, though not the direction of, 'variable-specific' interactive effects is a log-likelihood ratio test comparing the unrestricted and restricted models, where the restricted model omits [the product term]." Brambor, Clark, and Golder (2006, 77; see also Frant 1991) concur: "If one wants to test a conditional hypothesis in a meaningful way, then the analyst has to include an explicit interaction term..."

In this essay, we argue that these two prescriptions are inconsistent. In the BDV context, a significant product term coefficient is not necessary to confirm a hypothesis of contingent causality when the focus is on the marginal effects of independent variables on Pr(Y). We contend that findings of interaction between variables in influencing Pr(Y) based on a logit or probit model without any product terms are substantively meaningful and not merely an artifact of methodology.

The upshot of our argument is that, when employing a BDV, the prevailing consensus for statistically testing arguments of contingent causality should change. Fortunately, existing statistical techniques are useful for executing the prescriptions we will offer at the conclusion of our essay. Indeed, the plots that Brambor, Clark, and Golder (2006) recommend as diagnostics of interaction are still very helpful for detecting interaction effects that do not depend on a statistically significant product term. The CLARIFY simulation methodology proposed by King, Tomz, and Wittenberg (2000) is also useful for implementing our recommendations.

The remainder of the paper proceeds as follows. First, we review the rationale for the claim that a statistically significant product term estimate is essential for validating a hypothesis of interaction in a BDV setting. In the next section, we argue that a significant product term estimate is neither a necessary nor sufficient condition for claiming interaction among independent variables in influencing Pr(*Y*). Though a significant product term would be necessary to confirm interaction if *Y* were a continuous dependent variable, we argue that the analogy to the BDV case does not hold. The final sections offer recommendations for testing conditional hypotheses about binary dependent variables, which we illustrate by revisiting classic studies of the effect of voter registration restrictions on the probability of voting.

The Prevailing Case for Testing Conditional Hypotheses with Product Terms

The Structure of Binary Dependent Variable Models

Logit and probit models are designed to model a phenomenon of interest, *Y*, that is discrete: that is, it occurs (to be denoted event *Y*) or it does not. In both models, whether *Y* occurs has a Bernoulli distribution described by the parameter Pr(*Y*), the probability that the event occurs (Beck, King and Zeng 2000, 24). Pr(*Y*) is assumed to be determined by a set of independent variables $\{X_1, X_2, \ldots, X_k\}$, and a corresponding set of parameters $\{\beta_0, \beta_1, \beta_2 \ldots, \beta_k\}$, through a nonlinear link function, *G*, that maps the unbounded index $A = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_k X_k$ into the bounded probability space [0,1]:

$$\Pr(Y) = G(A)$$

In the logit model, the link function G is the logistic cumulative density function:

$$G(A) = \Lambda(A) = \frac{1}{1+\exp(-A)}$$

For the probit model, the Gaussian normal cumulative density function is used instead:

$$G(A) = \Phi(A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{A} \exp\left(-\frac{A^2}{2}\right) dA$$

These link functions are graphed in Figure 1, which shows their great similarity. Both map an A value of 0 into a probability of 0.5. Both are maximally sloped when Pr(Y) = 0.5, and decline in slope as Pr(Y) departs from 0.5 in either direction, approaching a slope of 0 as Pr(Y) approaches its floor of 0 or its ceiling of 1, and as A approaches ± ∞.

For both models, for any independent variable (say, X_1), one can determine the marginal (or instantaneous) effect of X_1 on Pr(Y) – $\partial \Pr(Y)/\partial X_1$ – at any set of values for the independent variables by calculating:

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\Pr(Y)}{\partial A}\right] * \left[\frac{\partial A}{\partial X_1}\right] \quad [1]$$

The first term on the right side of this equation is $\partial \Pr(Y)/\partial A$ – the marginal effect of the unbounded index A on Pr(Y). This marginal effect is the instantaneous slope of the logit or probit curve in Figure 1 at the values for the independent variables; it is maximized when A is zero and decreases as A departs from 0 in either direction. The second term on the right is $\partial A/\partial X_1$ – the marginal effect of X_1 on the unbounded index. Thus, in both the logit and probit models, the marginal effect of X_1 on Pr(Y) is a function of the marginal effect of X_1 on the unbounded index A, and the marginal effect of A on Pr(Y).

When the model includes no product term, equation 1 takes the form:

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\Pr(Y)}{\partial A}\right] * \left[\frac{\partial A}{\partial X_1}\right] = \left[\Lambda(A) * (1-\Lambda(A))\right] * \left[\beta_1\right] \qquad \text{(logit)}$$

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\Pr(Y)}{\partial A}\right] * \left[\frac{\partial A}{\partial X_1}\right] = \left[\Phi'(A)\right] * \left[\beta_1\right] \qquad \text{(probit)}$$

Note that in this model the marginal effect of X_1 on the unbounded index, $\partial A/\partial X_1$, is constant at β_1. But the marginal effect of X_1 on Pr(Y) depends on the value of all independent variables in the model, because ∂Pr(Y)/∂A is determined by the values of all independent variables.

Consider, for example, a simple logit model with constant term, – 4, and two independent variables, X_1 and X_2, both having β_j coefficients of 1:

$$\Pr(Y) = G(-4 + X_1 + X_2) \qquad [2]$$

In this model, the marginal effect of X_1 on A is constant at 1, but the marginal effect of X_1 on Pr(Y) varies with the values of X_1 and X_2. To illustrate, Figure 2A plots equation 2 for all values of X_1 in the range [0, 8] at three different values of X_2: 0, 1 and 2.

For any of the three curves, start at the point where Pr(Y) = 0.5. As X_1 moves in either direction from this point and pushes Pr(Y) closer to 1 or 0, X_1's marginal effect on Pr(Y)—the slope of the curve—decreases. This change in slope is an inherent feature of logit's functional form that serves to keep Pr(Y) within the [0, 1] boundaries of probability. For example, consider the right-most curve in Figure 2A. $\Pr(Y \mid X_1 = 7, X_2 = 0) \approx 0.95$. From here, even very large increases in X_1 (say, from 7 to 300) can cause at most a 0.05 rise in Pr(Y). By contrast, $\Pr(Y \mid X_1 = 4, X_2 = 0) = 0.5$. From here, even smaller increases in X_1 can cause a ten times larger increase in Pr(Y) simply because there is further room for Pr(Y) to rise; in fact, changing X_1 from 4 to 7 causes a 0.45 rise in Pr(Y). The same principle holds when X_1 is very small. $\Pr(Y \mid X_1 = 1, X_2 = 0) \approx 0.05$; from here, even huge reductions in X_1 cannot reduce Pr(Y) more than 0.05. The decline in the marginal effect of X_1 on Pr(Y) as this probability moves away from 0.5 is reflected in the S-shaped character of the curves in Figure 2A.

Note too that if we start at any point where Pr(Y) = 0.5, and adjust X_2 higher or lower (holding X_1 constant), the marginal effect of X_1 on Pr(Y) will also decline. For example, when X_1 = 4 and X_2 = 0, Pr(Y) = 0.5, and the marginal effect of X_1 on Pr(Y) is 0.25. If we leave X_1 at 4 but increase X_2 to 1, $\partial \Pr(Y)/\partial X_1$ drops to 0.197; if X_2 is increased one more unit to 2, the marginal effect declines to 0.105.

Thus, the marginal effect of X_1 on Pr(Y) depends on the values of both X_1 and X_2 in any logit or probit model; the marginal effect of X_1 on Pr(Y) is greatest when Pr(Y) is 0.5 and declines when a change in either variable pushes Pr(Y) toward 0 or 1. We refer to this feature of logit and probit as *compression*, because very large or small values of Pr(Y) compress further possible change in Pr(Y) to ever-smaller ranges. To be specific, a *compression effect* describes the phenomenon where, at high or low levels of Pr(Y), the marginal effect of any variable that has a strong effect on Pr(Y) when Pr(Y) is close to 0.5 declines appreciably in strength as Pr(Y) approaches 0 or 1. Compression effects will be present in any model that assumes that the link function mapping the index A into Pr(Y) is monotonic, including the logit and probit models.[2]

Assume, in contrast, that the logit model includes a product term, X_1X_2, so that $A = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_p X_1 X_2 + \beta_3 X_3 + \ldots + \beta_k X_k$. Now, equation 1 takes the form

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\Pr(Y)}{\partial A}\right] * \left[\frac{\partial A}{\partial X_1}\right] = \left[\Lambda(A) * (1 - \Lambda(A))\right] * \left[\beta_1 + \beta_p X_2\right] \qquad \text{(logit)}$$

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\Pr(Y)}{\partial A}\right] * \left[\frac{\partial A}{\partial X_1}\right] = \left[\Phi'(A)\right] * \left[\beta_1 + \beta_p X_2\right] \qquad \text{(probit)}$$

[2] To be more precise, compression is guaranteed only when the independent variables are unbounded. If these variables have finite ranges, Pr(Y) can stay within [0,1] even if the marginal effects of independent variables on Pr(Y) are linear and additive, so that the marginal effect of each variable remains unchanged as the values of the independent variables shift.

With the inclusion of the product term, the marginal effect of X_1 on the unbounded index ($\partial A/\partial X_1$)—while constant across all values of X_1—varies linearly with X_2: $\partial A/\partial X_1 = \beta_1 + \beta_p X_2$. Thus, the inclusion of the product term influences the marginal effect of X_1 on Pr(Y) in two ways: by changing the value of A (hence changing $\partial \Pr(Y)/\partial A$), and by adding the $\beta_p X_2$ term to $\partial A/\partial X_1$.

To illustrate the implications of a product term, consider a logit model that adds a X_1X_2 term to equation 2 with coefficient -0.30:

$$\Pr(Y) = G(-4 + X_1 + X_2 - 0.30X_1X_2)$$

Figure 2B plots the relationship between X_1 and Pr(Y) at various values of X_2 for this new model. We see that a compression effect is also present in this model; the marginal effect of X_1 on Pr(Y) is still greatest at Pr(Y) = 0.5, and declines when a change in either X_1 or X_2 pushes Pr(Y) toward either 0 or 1. However, the addition of the product term to the model introduces a second source of variation in the marginal effect of X_1 on Pr(Y), and in doing so, modifies the rate of compression in the marginal effect of X_1 on Pr(Y) as X_2 changes. For example, in both models, the marginal effect of X_1 on Pr(Y) is 0.105 when $X_1 = 6$ and $X_2 = 0$ (compare Figure 2A with Figure 2B.) Hold X_1 at 6, but increase X_2 to 2. In Figure 2A—the model without a product term—$\partial \Pr(Y)/\partial X_1$ declines appreciably to 0.018; in Figure 2B—the model with a product term—this marginal effect declines only slightly to 0.096.

The discussion above makes clear that for any two variables (e.g., X_1 and X_2) in a logit or probit model, the marginal effect of X_1 on Pr(Y) will vary with X_2, whether or not the model includes a product term, due to what we have termed a compression effect. When a X_1X_2 term is added to the model, there is an additional source of variation in the marginal effect of X_1 on Pr(Y) owing to the fact that $\partial A/\partial X_1$ is no longer constant.

Meaningful and Non-Meaningful Interaction

The prevailing consensus holds that a statistically significant product term coefficient estimate is necessary to confirm that X_1 and X_2 interact in influencing Pr(Y). This consensus rests on the contention that compression effects do not reflect substantively meaningful interaction between X_1 and X_2. Two reasons have been offered in the literature to support this view.

1. **Compression effects are an assumption of the logit and probit models, and not an underlying feature of the data.**

There have been several cases in which political scientists finding evidence of what we have termed compression effects in their logit or probit models have reported their results as indicative of substantively meaningful interaction, and been criticized. In one instance, Wolfinger and Rosenstone (1980) analyzed voter turnout in the 1972 presidential election using probit. These researchers were interested in, among other things, investigating the relationship between voter turnout and how easy it was for citizens to register to vote. By comparing the estimated marginal effect of voter registration provisions on the probability that someone will vote at different values of education, Wolfinger and Rosenstone (1980, 79) concluded that "[l]iberalizing registration provisions would have by far the greatest impact on the least educated and relatively little effect on well-educated people." As there were no product terms involving education and any of the voter registration provisions in their model, the decline in the marginal impact of voter registration provisions on the probability of voting as education increased must be what we have termed a compression effect, something the authors of the study understood.[3]

[3] Wolfinger and Rosenstone (1980, 11) write, "…the effect of a variable depends on the probability that the individual would vote. For example, a high-status occupation or a high income has less impact on a college graduate, who is 90 percent likely to vote, than it has on a high school dropout, who is only 55 percent likely to vote… With probit a variable has very little impact on those who are either very unlikely or nearly certain to vote. It has the greatest impact

In his 1991 *AJPS* piece, Nagler argues that Wolfinger and Rosenstone's claim of interaction "does not necessarily follow from [their] data analysis and is instead an artifact of the methodology used" (Nagler 1991, 1393):

> Since persons with the lowest education level are those who on average are closest to having .5 probability of voting, estimates of change based on that group will necessarily be larger than estimates of change for any other group. Thus, the smaller effects of [registration restrictions] on better-educated groups is simply an artifact of the changing slope of the normal density curve, not a result of a unique relationship between individuals' education and state registration requirements. Hence, to infer substantive interaction between registration laws and education based on the differing estimated impacts is incorrect. This interaction is *assumed* by the model specification (Nagler 1991, 1397).

A similar exchange occurred between Berry and Berry (1990) and Frant (1991). Berry and Berry published conclusions that independent variables interact in influencing the probability of a lottery adoption by an American state based on results from a probit model including no product terms. Frant argued that Berry and Berry's interaction findings were not substantively meaningful:

> [Berry and Berry] present as empirical results what are in reality artifacts of the way the model is specified... They find, for example, that the effect [on the probability of a lottery adoption] associated with an election year is greater (in absolute terms) when the state is in poor fiscal health than when it is in good fiscal health. This sounds like a *conclusion* (and the authors treat it as such); but actually, it is an *assumption* of the model. …This is a plausible assumption, but it is only an assumption. When we employ a probit (or logit) model, this assumption is built in (Frant 1991, 572).

Nagler's and Frant's positions reflect the prevalent message in the political methodology literature that any compression effect detected by a logit or probit model without a product term is an artifact of the estimation procedure, and not a substantively meaningful phenomenon that should be interpreted as causal interaction. Recently, Brambor, Clark, and Golder (2006, 77) advised readers that compression effects occur "whether the analyst's hypothesis is conditional

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in the middle of the distribution, on those who...are most susceptible to the forces pushing them to vote or not to vote."

or not—it is just part and parcel of deciding to use a nonlinear model such as probit; it is always there. If one wants to test a conditional hypothesis in a meaningful way, then the analyst has to include an explicit interaction term…" (see also Huang and Shields 2000).

2. **Compression effects do not reflect the interactive relationship between two variables, because in a multivariate model they occur between all the independent variables. Product terms, by contrast, capture this bivariate relationship.**

Compression effects exist because, as Pr(Y) approaches its limits of 1 and 0, even powerful causal variables cannot increase/decrease the probability of an event beyond the upper/lower limit that probabilities can theoretically reach. In a multivariate model, where multiple causal factors influence Pr(Y), these limits may be approached via the influence of varying combinations of causes. Suppose, for example, that Y is influenced by three variables, X_1 X_2 and X_3, as in the following logit model:

$$\Pr(Y) = G\left(X_1 + 0.5X_2 + 0.25X_3\right)$$

In this model, when $X_1 = X_2 = X_3 = 0$, the index $A = X_1 + 0.5X_2 + 0.25X_3$, equals 0, a value for the index that is transformed by the logit link function to Pr(Y) = 0.5. At this point, the marginal effect of X_1 on Pr(Y) is maximized at 0.25. If X_1 and X_3 are left at 0 but X_2 is increased to 4.5, the unbounded index A grows to 2.25, at which point Pr(Y) ≈ 0.905, and the marginal effect of X_1 on Pr(Y) declines to 0.086. Thus, the compression effect in the logit model makes it so that the marginal effect of X_1 on Pr(Y) declines by 0.164 (= 0.25 – 0.086) as X_2 increases from 0 to 4.5 while X_1 and X_3 are held constant. One might therefore claim that X_1 and X_2 interact in influencing Pr(Y). But one could also claim that X_1 interacts with X_3 in influencing Pr(Y) using identical logic: starting at $X_1 = X_2 = X_3 = 0$, if X_1 and X_2 are held constant but X_3 is increased to 9, the index A also grows to 2.25, once again Pr(Y) ≈ 0.905, and the marginal effect of X_1 on Pr(Y) declines to 0.086.

The consensus view seems to be that, because compression effects force the marginal effect of X_1 on Pr(Y) to vary with the value of every independent variable in the model, researchers should not interpret compression effects as relevant to causal interaction between any specific pair of variables. Nagler invokes this criticism to challenge Wolfinger and Rosenstone's (1980) claim that voter registration provisions interact with education in influencing the probability of voting. He writes:

> ...to the extent that those with less education are nearest to the .5 mark in their expected probability of voting (or become nearest to the .5 mark in their expected probability of voting after altering a different independent variable), they will be more affected than those with higher education levels by changes in *any* independent variable (Nagler 1994, 1397).

Thus, Nagler asserts that compression effects cannot tell us about interaction that takes place between a specific pair of variables. The only substantively important causal interaction between two variables is what he calls a "variable-specific interactive effect" (Nagler 1994, 249). He makes clear his belief that product terms are needed to capture this kind of interaction:

> If adding the [product term] to the set of variables $X_1,\ldots, X_k$, leads to an improvement in the model, then I would argue that we have "variable-specific" interaction between [the two variables], as opposed to interaction imposed between all the variables by the functional form of the model (Nagler 1994, 249-50).

Since the publication of Nagler's 1991 paper, political scientists have rarely, if ever, published claims that variables interact in influencing the probability that an event will occur based on a logit or probit model estimated without a product term. This suggests that the political science community has been convinced that compression effects do not reflect substantively meaningful interaction, and that as a result, a statistically significant product term coefficient estimate is necessary to confirm that X_1 and X_2 interact in influencing Pr(Y).

Product Terms and Substantively Significant Interaction

Contrary to the prevailing consensus, we argue that compression effects in the BDV setting are a substantively relevant form of interaction. When they are present, they are a fundamental feature of the data generating process (DGP) and highly meaningful. Indeed, compression effects are already routinely treated as meaningful by political scientists analyzing BDV models when they employ the widely-used tools for model interpretation that King, Tomz and Wittenberg (2000) and Brambor, Clark, and Golder (2006) have developed. When compression occurs, it is essential to an accurate description of how the marginal effect of one variable, X_1, on Pr(Y) varies with the value of another variable, X_2, and thus to a valid test of a hypothesis that X_1 and X_2 interact in influencing Pr(Y). Even when product terms are also needed to completely capture the interaction between two variables—a situation that sometimes, but not always occurs—compression effects cannot be ignored, as they still contribute important information. Our argument can be summarized into eight points, each of which we discuss in depth in this section.

1. **Compression effects should not be viewed as causally-irrelevant artifacts of logit and probit models. Compression effects constitute substantively meaningful interaction between independent variables in their joint causal influence on Pr(*Y*).**

Much of the discussion on interaction in BDV models has centered on the claim that compression effects are an artifact of the logit and probit specifications. It is apparently overlooked that logit and probit are recognized as an improvement over other models—particularly ordinary least squares linear probability models (LPMs)—precisely *because* they impose compression. Logit and probit are often a better match to a BDV causal process than the LPM because compression is intrinsic to many BDV causal processes, something for which the LPM does not account.

For any discrete event Y, Pr(Y) is bounded between 0 and 1. These boundaries should not be viewed as an assumption imposed on Pr(Y) by any specific statistical model (such as logit or probit). Pr(*Y*) values of 1.5 or -0.3 are nonsensical, as an event cannot occur 150 times out of 100 or negative 30 times out of 100. For this reason, even a powerful causal variable cannot push Pr(*Y*) above 1 or below 0, and any statistical model of the causal process that does not reflect this restriction must fail to describe the process correctly. In other words, any statistical model that can yield an estimated Pr(*Y*) outside of [0,1] must *ipso facto* misspecify the true DGP. LPMs do not restrict Pr(*Y*) to [0,1], and therefore yield nonsense predictions when causal variables are unbounded. Logit and probit, by contrast, force the marginal effect of independent variables to decline as Pr(*Y*) nears its limits, avoiding these meaningless predictions.

The 0-1 boundaries on probability can be a strong theoretical rationale for expecting variation in the marginal effect of a variable on Pr(*Y*). A foundational econometrics text often used in our discipline speaks to this point at length in discussing influences on the probability of home ownership:

> …the fundamental problem with the LPM is that it is not logically a very attractive model because it assumes that [Pr(*Y* | *X*)] increases linearly with *X*, that is, the marginal...effect of *X* remains constant throughout. Thus, in our home ownership example we found that as [income] increases by a unit ($1000), the probability of owning a house increases by the same constant amount of .10. This is so whether the income level is $8000, $10,000, $18,000, or $22,000. This seems patently unrealistic. In reality, one would expect… at very low income a family will not own a house but at a sufficiently high level of income, say *X**, it most likely will own a house. Any increase in income beyond *X** will have little effect on the probability of owning a house. Thus, at both ends of the income distribution, the probability of owning a house will be virtually unaffected by a small increase in *X* (Gujarati 1995, 552-53).

Similarly, Wolfinger and Rosenstone rely on the boundaries of probability to defend their proposition that all influences on an individual's decision to vote are conditional on the individual's level of education. For example, the authors write, "...a high-status occupation or a high income has less impact on a college graduate, who is 90 percent likely to vote, than it has on a high school dropout, who is only 55 percent likely to vote" (Wolfinger and Rosenstone

1980, 11). These variables have "very little impact on those who are either very unlikely or nearly certain to vote. [They have] the greatest impact in the middle of the distribution, on those who are between 40 and 60 percent likely to vote and are most susceptible to the forces pushing them to vote or not to vote" (Wolfinger and Rosenstone 1980, 11).[4]

2. Contrary to the claim that compression effects are an artifact of the logit and probit functional form, when appreciable compression effects are *not* present in the data generating process, logit and probit will *not* mistakenly detect them.

Nagler (1994) and Frant (1991) argue that compression effects detected via logit and probit should not be viewed as substantively meaningful interaction among variables in influencing Pr(*Y*) because compression is inherent in the functional form of logit and probit, and thus an artifact of the estimation procedure. It is true that with any logit or probit model, if independent variables are unbounded, a sufficiently large change in *any* independent variable having a nonzero coefficient will move Pr(*Y*) close enough to one of its boundaries that the possible marginal impact of any other independent variable will be severely restricted.

But in any real-world research setting, there are theoretical and practical limits on the range of independent variables. When independent variables are constrained to finite ranges, the causal process generating the probability of an event can, *but need not*, be characterized by compression. Consider, for example, the two-independent variable causal process reflected in the logit equation plotted in Figure 3A, in which the population values for X_1 and X_2 are

[4] Huang and Shields (2000, 81) call what we term the compression effect of logit and probit "built-in interaction." They share our view that when logit or probit models accurately specify the DGP, built-in interactions/compression effects should be viewed as meaningful, as they constitute one component of the total interaction among the independent variables. Huang and Shields (2000, 81) differ from us in that they agree with Nagler that claims of interaction in a logit or probit model "must be modeled by a statistically significant multiplicative term in the equation."

constrained to the [0,8] range.[5] In this DGP, there is substantial compression, as evidenced in the greater rate of ascent of the surface near its center than when both X_1 and X_2 assume their lowest values, or when both variables are at their highest. The DGP in Figure 3B, in which population values for X_1 and X_2 are also constrained to the [0,8] range, is a logit equation as well.[6] But despite the fact that this DGP is a logit equation, the surface is very nearly flat.[7] Thus, in the case of this logit model, it would be reasonable to claim that the effects of X_1 and X_2 on Pr(Y) are essentially linear and additive, and that very little compression is present.

Is there any reason to fear that, if we estimated a logit model with independent variables X_1 and X_2 using a data set generated from the DGP represented in Figure 3B, logit will lead us to conclude there is substantial compression even though very little is actually present? The answer must be no. It is well known that logit yields consistent parameter estimates (Aldrich and Nelson 1984). Consistent parameter estimates will generate consistent estimates of the marginal effects of independent variables on Pr(Y). Thus, with a sufficiently large and representative sample, estimated marginal effects on Pr(Y) will be on target. Consequently, if the true marginal effects of X_1 and X_2 are nearly constant over the range of values for X_1 and X_2 in the population, estimated marginal effects will also be nearly constant.

In sum, logit and probit models will not manufacture substantial compression where little is present as long as the model accurately specifies the data generating process. If there is only slight compression over the range of relevant values for the independent variables, logit and probit will not detect substantial compression. These models will detect meaningful

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[5] The logit model graphed is $\Pr(Y) = G\,(-8 + X_1 + X_2)$.

[6] The logit model plotted is $\Pr(Y) = G\,(-1 + 0.125X_1 + 0.125X_2)$.

[7] When both X_1 and X_2 are in the [0,8] range, $\partial \Pr(Y)/\partial X_1$ is always between 0.025 and 0.031.

compression only when there is meaningful compression in the DGP. An assertion to the contrary amounts to a claim that logit and probit yield inconsistent parameter estimates in accurately specified models, which clearly is not the case.

3. Most researchers studying BDVs in recent years routinely treat compression effects as substantively meaningful interaction, even if they do not explicitly recognize that they are doing so.

The current conventional approach to interpreting a binary logit or probit model—with or without a product term—is to use the MLEs for the model to derive estimates of the effects of variables on Pr(*Y*) using simulated first differences (King, Tomz, and Wittenberg 2000) or by plotting marginal effects (Brambor, Clark and Golder 2006). Sometimes scholars who are not positing any interaction use logit or probit MLEs to estimate the relationship between an independent variable and Pr(*Y*) when all other independent variables are held at central values (e.g., Lektzian and Sprecher 2007; Cunningham 2006). Scholars who posit interaction use logit or probit MLEs to estimate how the effect of one variable on Pr(*Y*) varies with the value another independent variable when remaining variables are fixed at central values (e.g., Gartzke 2007; Shipan and Volden 2006). Either way, as long as the logit or probit model accurately specifies the true DGP, the resulting estimates of marginal effects of variables on Pr(*Y*) will reflect whatever compression effect is present.

Thus, researchers reporting estimates of the marginal effects of variables on Pr(Y)—regardless of whether they posit interaction, and regardless of whether they include product terms in the model—are implicitly recognizing compression effects as substantively meaningful, because if the true DGP involves nontrivial compression, the reported effects on Pr(*Y*) will reflect this compression. We applaud the recent tendency for political scientists studying BDVs to heed King, Tomz and Wittenberg's (2000) advice to report effects of independent variables on a substantively meaningful dependent variable, such as the probability that an event will occur. We simply note that by doing so, scholars have tacitly acknowledged that compression effects are substantively relevant.

4. **In a logit or probit model, a statistically significant product term coefficient is not a *necessary* condition for substantial interaction among independent variables in their effect on Pr(*Y*).**

If a large product term coefficient were necessary for substantial interaction among independent variables in influencing Pr(Y), then a logit or probit model with no product term (i.e., one in which the product term's coefficient is constrained to be zero) would never exhibit substantial interaction among variables in their effects on Pr(Y). But our earlier examination of the model depicted in Figure 2A made clear that even in a model without a product term, compression can lead to substantial variation in the marginal effect of one variable on Pr(Y) across different values of another variable. For example, when $X_1 = 6$, a shift in X_2 from 0 to 2 reduces the marginal effect of X_1 on Pr(Y) from .105 to .018. Thus, a small coefficient for a product term does not guarantee the absence of substantial interaction along independent variables in influencing Pr(Y), and consequently, a statistically insignificant product term coefficient should not be taken as evidence of the absence of interaction in influencing Pr(Y).

5. **In a logit or probit model, a statistically significant product term coefficient is not a *sufficient* condition for substantively meaningful interaction between independent variables in their effect on Pr(*Y*) at all values of the independent variables.**

Consider the logit model of Figure 2B, which includes a product term of nontrivial magnitude. Does this large product term coefficient guarantee that we will find appreciable interaction between X_1 and X_2 in influencing Pr(Y) throughout the range of substantively meaningful values for X_1 and X_2? The figure shows clearly that it does not.

There are many values for X_1 and X_2 at which there is strong interaction between X_1 and

X_2. For instance, when $X_1 = 4$, the slopes of the three curves are quite different ($\partial \Pr(Y)/\partial X_1$ = 0.25, 0.173 and 0.096, respectively) indicating that the marginal effect of X_1 on Pr(Y) changes appreciably as X_2 rises from 0 to 2. But there are some values for the independent variables at which there is little interaction between X_1 and X_2. Indeed, if in the population of interest both X_1 and X_2 were constrained to the [0, 2] range, the three curves are close enough to being parallel that many analysts would be comfortable rejecting an hypothesis of interaction and concluding that the effects of X_1 and X_2 are approximately additive despite the large negative coefficient for the product term. When $X_1 = 1$, for example, the slopes of the three curves are very similar ($\partial \Pr(Y)/\partial X_1$ = 0.045, 0.058, and 0.056, respectively.)

6. In a logit or probit model, the sign of the coefficient for a product term may give a misleading signal about the "direction" of the interaction between independent variables in influencing Pr(*Y*). Put differently, the direction of interaction among independent variables in influencing the unbounded index *A* can be different from the direction of interaction in influencing Pr(*Y*).

By the "direction" of interaction between X_1 and X_2, we refer to whether the marginal effect of X_1 on the dependent variable increases or decreases in magnitude as X_2 increases. Since the logit model of Figure 2B has a negative coefficient for X_1X_2, the marginal effect of X_1 on the unbounded index A declines as X_2 increases regardless of the values for X_1 and X_2. However, the figure shows clearly that that there are values of X_1 and X_2 at which the marginal effect of X_1 on Pr(Y) *increases* as X_2 increases. This is true when $X_1 = 7$, at which $\partial \Pr(Y)/\partial X_1$ = 0.045, 0.079, and 0.086 when X_2 = 0, 1, and 2, respectively.

This phenomenon can be explained by examining the second derivative $\partial^2 \Pr(Y)/\partial X_1 \partial X_2$, which can be interpreted as the marginal effect of X_2 on the marginal effect of X_1 on Pr(Y). For a logit model, as Norton, Wang and Ai (2004, 158) have shown,

$$\partial^2 \Pr(Y)/\partial X_1 \partial X_2 = \Pr(Y)\,(1-\Pr(Y))\beta_p + [\Pr(Y)\,(1-\Pr(Y))(1-2\Pr(Y))(\beta_1 + \beta_p X_2)(\beta_2 + \beta_p X_1)]$$

Thus, the direction and magnitude of interaction between X_1 and X_2 in influencing Pr(Y) is affected not only by the sign of product term coefficient (β_p), but by the values of X_1 and X_2, and the coefficients for these variables.

7. **The fact that in a logit or probit model, the marginal effect of an independent variable on Pr(*Y*) varies with the values of all variables in the model does not invalidate a claim that two variables interact even if there is no product term in the model.**

One argument for not regarding compression as meaningful interaction is that the variation in the marginal effect of a variable associated with compression is not variable-specific (Nagler 1994). That is, the marginal effect of a variable X_1 on Pr(Y) will change when any variable in the model changes. Therefore, we ought not interpret the change in marginal effect as some other variable changes as substantively meaningful, and claim the presence of interaction.

We contend that this position falsely equates (for a model with k independent variables) the hypothesis that X_1 and X_2 interact with the hypothesis that X_1 does *not* interact with any of X_3, X_4,..., X_k. In fact, advancing the former hypothesis does not imply a belief that the latter is true. To continue with the voting turnout example of Wolfinger and Rosenstone, a belief that high levels of education desensitize a person to changes in voter registration requirements (because highly educated people are already very likely to vote) does *not* imply that high or low values of other variables (e.g., a high age) do not cause a similar desensitization (because senior citizens are also highly likely to vote).

Indeed, even when all variation in the marginal effects of variables on Pr(Y) is due to compression—as in a model with no product term—an analyst interested primarily in the effects of X_1 and X_2 is perfectly justified in measuring and reporting how the marginal effect of X_1 on Pr(Y) varies with the value of X_2 when X_3,..., X_k are held constant at specified values (e.g., their mean). This constitutes an accurate reflection of the influence of X_1 and X_2 on Pr(Y) in the context defined by the specified values for $X_3,\ldots, X_k$. Of course, any such report is contingent on the values at which $X_3,\ldots, X_k$ are fixed, and different contextual values for these variables will produce different results.

8. Product terms still have an important role to play in empirical research using logit and probit.

Though we have stressed that compression effects are substantively significant when researchers focus on the effects of variables on Pr(Y), as they have been urged to do by much of the methodological literature, we believe that product terms are still appropriate when a researcher believes that there is interaction between two independent variables in influencing logit's or probit's unbounded index, A. Indeed, there are cases where A is theoretically meaningful and thus a quantity of interest itself.

For example, when the logit or probit model is specified because the underlying DGP is a utility maximization model over a binary choice, A corresponds to the systematic difference in utility between the two possible choices (McFadden 1973). One theory of election turnout says that a citizen votes when the expected utility of voting is greater than the expected utility of not voting, i.e., $E[U_{\text{vote}}] - E[U_{\text{no vote}}] > 0$. If this difference in utility is subject to an error component, ε, then a citizen votes whenever

$$E[U_{\text{vote}}] - E[U_{\text{no vote}}] + \varepsilon > 0,$$

or alternatively,

$$E[U_{\text{no vote}}] - E[U_{\text{vote}}] < \varepsilon.$$

$\Pr(E[U_{\text{no vote}}] - E[U_{\text{vote}}] < \varepsilon)$ is the probability that a voter turns out at the polls. The distribution of ε determines the form of the appropriate statistical model; a Gaussian normal ε implies probit, while a logistic ε implies logit. The quantity $E[U_{\text{no vote}}] - E[U_{\text{vote}}]$ corresponds to the index $A = \beta_0$

$+ \beta_1 X_1 + \beta_2 X_2 + ... + \beta_k X_k$, where observable X variables (like education or voting registration laws) are influences on the size of this difference in utility.

If (in this substantive example) A includes a X_1X_2 term that proves to be statistically significant, then it would be appropriate to infer that X_1 and X_2 interact in influencing the utility that a citizen derives from voting. In this case, the researcher can justifiably use the MLE of the product term coefficient as a point estimate of the magnitude of interaction between X_1 and X_2 in influencing the utility from voting. However, a researcher cannot draw inferences about interaction between X_1 and X_2 in influencing the *probability* of voting directly from the MLEs for coefficients; she must use the MLEs to compute estimated marginal effects on this probability and perform appropriate CLARIFY analyses or construct relevant marginal effects plots. The estimates of the marginal effects of X_1 and X_2 on the probability of voting so obtained would incorporate not only interaction between X_1 and X_2 in influencing the utility of voting, as reflected in the MLE for the X_1X_2 coefficient, but whatever compression effect is present in the DGP as well.

Deciding Whether to Include a Product Term

We have argued that in a logit or probit model, a statistically significant product term coefficient is neither necessary nor sufficient for concluding that there is substantively meaningful interaction among independent variables in their influence on Pr(Y). How, then, should a researcher interested in determining whether there is interaction among independent variables in influencing Pr(Y) decide whether to include a product term in a logit or probit model? Assume that logit or probit properly specifies the theory being tested, and consequently that theoretical guidance for the functional form of $A = \Sigma\beta_i X_i$ exists. Such guidance would certainly be present if *(i)* the hypothesis about interaction among the independent variables in influencing Pr(*Y*) is motivated by a theory positing the existence of an unbounded latent variable, *A* (e.g., a utility maximization model), (*ii*) the observed BDV is assumed to be a realization of *A*, and (*iii*) a priori theory leads to a clear prediction about the functional form of $A = \Sigma\beta_i X_i$. If theory predicts that the independent variables interact in influencing the unbounded latent variable *A*, a logit or probit model specifying the interaction with one or more product terms should be estimated, and the standard statistical test proposed by Nagler (1991) should be employed to test for interaction.[8] To determine whether this interaction extends to the variables' influence on Pr(*Y*), the researcher should rely on marginal effects plots and/or CLARIFY analysis.

On the other hand, the researcher's theory may give no reason to believe that the effects of independent variables on the latent variable *A* are interactive. In this case, there is no need to include product terms in the model. If there is doubt or competing theoretical guidance, the proposition that effects on *A* are additive can be tested by including one or more product terms in the model, terms expected to have parameter values of zero, and determining whether their coefficient estimates are nearly zero. Even if there is no interaction among independent variables in influencing the unbounded latent variable, CLARIFY analysis and/or marginal effects plots should be used to determine whether compression effects are causing interaction among independent variables in influencing Pr(*Y*).[9]

[8] Indeed, since the latent variable *A* is unbounded, one can interpret estimated logit or probit coefficients in much the same way as the coefficients in an OLS regression model with a continuous dependent variable. For example, the coefficient for X_1 (in a model including X_1, X_2 and X_1X_2) represents the marginal effect of X_1 on *A* (at any value of X_1) when X_2 equals 0. For good discussions of interpretations of regression models with product terms, see Friedrich (1982), Braumoeller (2004) and Brambor, Clark and Golder (2006).

[9] A Stata program designed by Brambor, Clark and Golder to construct marginal effects plots for logit and probit models including a product term (available at

An Applied Empirical Demonstration of an Appropriate Test for Interaction

To illustrate our recommendations, we examine the relationship among education, voter registration provisions and the probability of voting in a presidential election (to be denoted Pr(*vote*)) initially investigated by Wolfinger and Rosenstone (1980), and later analyzed by Nagler (1991, 1994) and Huang and Shields (2000); we consider the hypothesis that education and voter registration provisions interact in influencing Pr(*vote*). We employ 1984 Current Population Survey data (n = 99,676) from a replication data set for Nagler's (1994) study provided by Altman and McDonald (2003). The operational definition of voter registration provisions is *closing date*, the number of days before an election at which registration is closed.[10] The variable *education* measures number of years of schooling collapsed into 8 categories, coded 1 (0 to 4 years), 2 (5-7 years), 3 (8 years), 4 (9-11 years), 5 (12 years), 6 (1-3 years of college), 7 (4 years of college), and 8 (5 or more years of college). Control variables include a respondent's *age* in years, whether the respondent lives in the *south*, and whether a *gubernatorial election* occurred in the state during the presidential election year.

Wolfinger and Rosenstone (1980) test the proposition that *education* and *closing date* interact in influencing Pr(*vote*) using a probit model without product terms; Nagler (1991) tests the hypothesis with a probit model including product terms involving voter registration provisions and education: *closing date***education* and *closing date***eduation*^{2}. Which specification is correct hinges on the theory underlying the empirical analysis: only theory about the effects of these independent variables on a latent unbounded variable for which the BDV is

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http://homepages.nyu.edu/%7emrg217/interaction.html) can be modified for use with models without a product term.

[10] Wolfinger and Rosenstone originally included additional operationalizations of voter registration provisions that are excluded in the Nagler data, and therefore excluded from our analysis.

assumed to be an indicator (such as the utility of voting) can reasonably guide the choice about whether to include product terms.

Though they do not explicitly offer these theories, Nagler's and Wolfinger and Rosenstone's specifications are consistent with different utility maximization theories about the influence of education and voter registration provisions on the utility of voting. Wolfinger and Rosenstone's specification is consistent with a theory that the independent variables in the model have additive effects on the (unbounded) utility of voting, and thus that any interaction between education and voter registration provisions on the (bounded) probability of voting results from compression. That is, Wolfinger and Rosenstone's specification implies that the utility highly educated people derive from voting is already so likely to be above the threshold necessary to justify voting that additional benefit from easier voting registration will probably not make these people more likely to vote. In this explanation, people at all levels of education derive the same utility from relaxing voter registration restrictions, but only the behavior of less-educated people is likely to be affected because these are the people who are most likely to be nearly indifferent between voting and not voting. In contrast, Nagler's specification is consistent with a theory that the independent variables interact in their effect on the latent utility of voting, such that the effect of voter registration restrictions on an individual's utility of voting varies depending on her level of education.

Table 1 presents parameter estimates from a probit model reflecting Nagler's specification in column 1. The coefficient estimate for *closing date*education* is negative and statistically significant (p = .03); the MLE for *closing date*eduation*^{2} is positive, but barely misses significance at the .05 level (p = .06). This constitutes empirical evidence that there is interaction between *closing date* and *education* in influencing the unbounded utility of voting, such that the negative impact of restrictive voter registration rules on the utility derived from voting becomes stronger as level of education increases.[11] Yet the fact that both product term coefficient estimates have *p* values between .03 and .06 with a sample size as large as 99,000 suggests the possibility that the magnitude of these coefficients is small, and thus the interaction between education and voter registration restrictiveness in influencing the utility of voting is weak (see note 12).

Figure 4 presents a plot of the estimated marginal effect (along with confidence bands) of *closing date* on the probability of voting at different levels of *education* while holding other continuous variables at their mean and other non-continuous variables at their mode. One can see that strict registration provisions have their strongest effect in suppressing the probability of voting among individuals with an *education* score of 5, which indicates a high school degree. At this education level, the point estimate of the marginal effect of *closing date* on Pr(*vote*) is -0.0031. This marginal effect implies that an increase of one week in the date prior to the election that registration closes prompts a decrease of approximately .022 (= 7*0.0031) in the probability of voting. As *education* increases or decreases from a value of 5, the marginal effect of *closing date* subsides. For someone with the lowest *education* score of 1 (0-4 years of schooling), the marginal effect is -0.0008; when *education* is maximized at 8 (5 or more years of college), the marginal effect is -0.0010.[12]

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[11] The coefficients for *closing date*, *closing date***education* and *closing date***eduation*^{2} together imply a point estimate for the marginal effect of *closing date* on the latent utility of voting of – 0.0023 when *education* is at its lowest (1) and – 0.0071 when *education* is at its highest (8).

[12] We also estimated the probit model deleting both product terms, and constructed a plot of marginal effects like that in Figure 4. It has the same basic U-shape as Figure 4, and leads to a similar substantive conclusion. (This figure is included in the replication data set available at http://www...edu.) [Note to reviewers: This figure is appended as the last page of this manuscript.] Thus, the inclusion of the product terms in the model has very little impact on

We also calculate relevant first differences in Pr(*vote*) using CLARIFY. We find that for someone with a high school degree (*education* = 5), the probability of voting rises 0.072 when the stringency of voter registration provisions is relaxed from its mean (registration closes 24.8 days before the election) to 0 (registration is allowed right up to the day of the election), while other variables are fixed at central values; this change score has a 95% confidence interval of [0.065, 0.080]. By contrast, the same probability difference for those with the least education (0-4 years of schooling) is 0.019 (CI = [–0.019, 0.060]). Furthermore, the difference between these two probability differences is 0.053 (= 0.072 – 0.019), with a confidence interval of [0.016, 0.088]. Since this confidence interval excludes zero, we can reject the null hypothesis that the effect of voter registration provisions on the probability of voting is the same at low and middling education levels. For those who are most educated (5 or more years of college), the first difference in probability of voting when moving from mean to minimum stringency of registration provisions is 0.022 (CI = [0.013, 0.030]). The difference between this probability difference and the one for those with a high school degree is 0.050 (= 0.072 – 0.022), with a confidence interval [0.040, 0.062]. Hence, we can also reject the null hypothesis that the effect of *voter registration provisions* on the probability of voting is the same at high and middling levels of education.[13]

Thus, there is considerable statistical evidence of interaction between voter registration provisions and education in influencing the probability of voting such that when the remaining

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estimated marginal effects on the probability of voting, confirming our earlier speculation that the product term coefficients have small magnitude.

[13] Using 1972 data, Huang and Shields (2000, Figure 1) present an analysis of predicted probabilities of voting that leads to the same general conclusions as our CLARIFY analysis. However, working prior to the introduction of CLARIFY, they derived predicted probabilities analytically, rather than by simulation, and did not present confidence intervals for estimated probabilities.

independent variables assume central values, the effect of restrictive registration provisions in suppressing the probability that an individual will vote is strongest at middling levels of education and considerably weaker at both high and low education levels. Since the coefficient estimates for the only two product terms in the model are small in magnitude, the interaction is due primarily to compression. For persons who are already either highly likely to vote or highly unlikely to vote, registration restrictions have a relatively weak effect on the probability of voting. The impact of restrictions is more powerful on individuals whose probability of voting is nearer to 0.50.

Conclusion

In this paper, we have argued that one can glean very little definitive information about the nature of interaction among independent variables in influencing Pr(Y) from the sign and magnitude of a product term coefficient in a logit or probit model. Depending on the values of interest for the independent variables, (*i*) there can be substantial interaction among independent variables in influencing Pr(Y) even when the coefficient for all product terms is zero, (*ii*) there can be little interaction among independent variables in influencing Pr(Y) even when product term coefficients are large, and (*iii*) when there is both strong interaction among independent variables in influencing Pr(Y) and large product term coefficients, the interaction among variables in effects on Pr(Y) can be in a different direction than their interaction in influencing the unbounded index A. Testing the statistical significance of the MLEs for product terms is necessary to confirm a hypothesis that independent variables interact in influencing the unbounded index, and when the index is itself of theoretical interest, this is a meaningful exercise. But this test does not shed light on the nature of the interaction between the variables in influencing Pr(Y). Whether the variables interact in influencing Pr(Y) should be tested by direct examination of estimated effects on Pr(*Y*) using CLARIFY or marginal effects plots like those suggested by Brambor, Clark and Golder (2006).

We conclude with a warning: all the recommendations we advance in this essay are contingent on logit or probit being the correct functional form for the data generating process being studied. Although logit and probit are functional forms that can accommodate a variety of DGPs, they are still quite restrictive (Beck, King and Zeng 2000). If the true DGP is misspecified by logit or probit, then estimates of the model's parameters (and any marginal effects derived there from) will be biased and inconsistent. This is true whether the model includes or excludes product terms. However, it is possible that even when the true functional form for a DGP is neither logit nor probit, these models may approximate the DGP closely enough to yield estimates of the marginal effects of variables on Pr(*Y*) that are close to their true population values. In a companion paper (Authors 2006), we use Monte Carlo methods to investigate how well logit and probit—with and without product terms—perform in recovering "true" marginal effects when the true functional form is neither logit nor probit.[14]

[14] Note to editor and reviewers: The concluding sentences will need to be revised depending on the publication status of the companion paper, which is currently being revised for submission to a journal. But the points made in the manuscript you are reviewing stand independent of analyses reported in the companion paper.

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Table 1. Probit Models of 1984 Presidential Voting Turnout

Independent Variable	Dependent Variable: Pr(*vote*) (1) Model with Product Terms	(2) Model without Product Terms
closing date	0.0006	-0.0078*
	(0.0037)	(0.0004)
education	0.2645*	0.1819*
	(0.0416)	(0.0144)
education^2	0.0051	0.0123*
	(0.0042)	(0.0014)
age	0.0697*	0.0697*
	(0.0013)	(0.0013)
age^2	-0.0005*	-0.0005*
	(0.0000)	(0.0000)
south	-0.1155*	-0.1159*
	(0.0110)	(0.0110)
gubernatorial election	0.0034	0.0034
	(0.0116)	(0.0116)
*closing date*education*	-0.0032*	
	(0.0015)	
*closing date*education*^{2}	0.00028	
	(0.00015)	
Constant	-2.7431*	-2.5230*
	(0.1074)	(0.0486)
N	99676	99676
Log-likelihood	-55815.28	-55818.03

NOTE: z-scores in parentheses; * p ≤ .05.

Figure 1. The Logit and Probit Link Functions

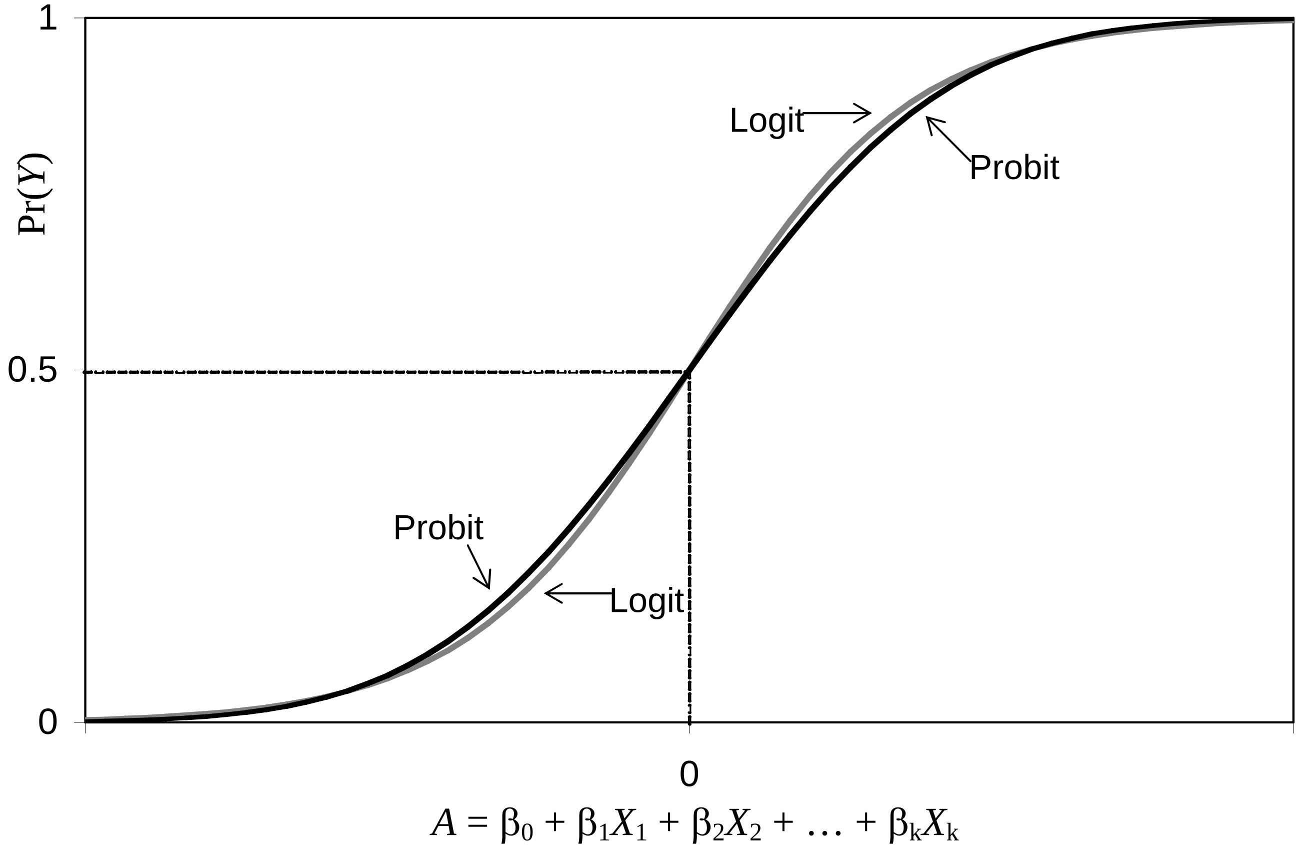
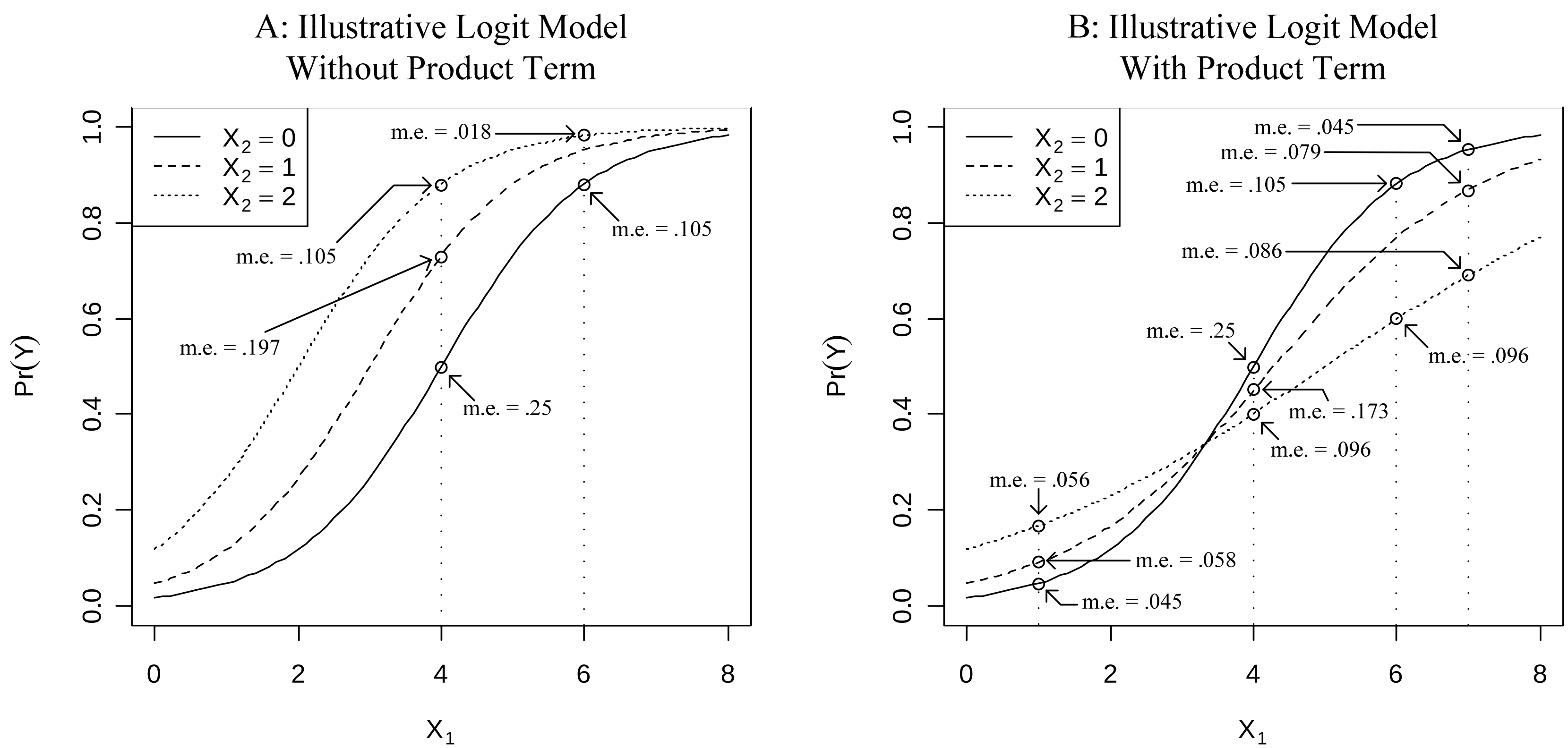
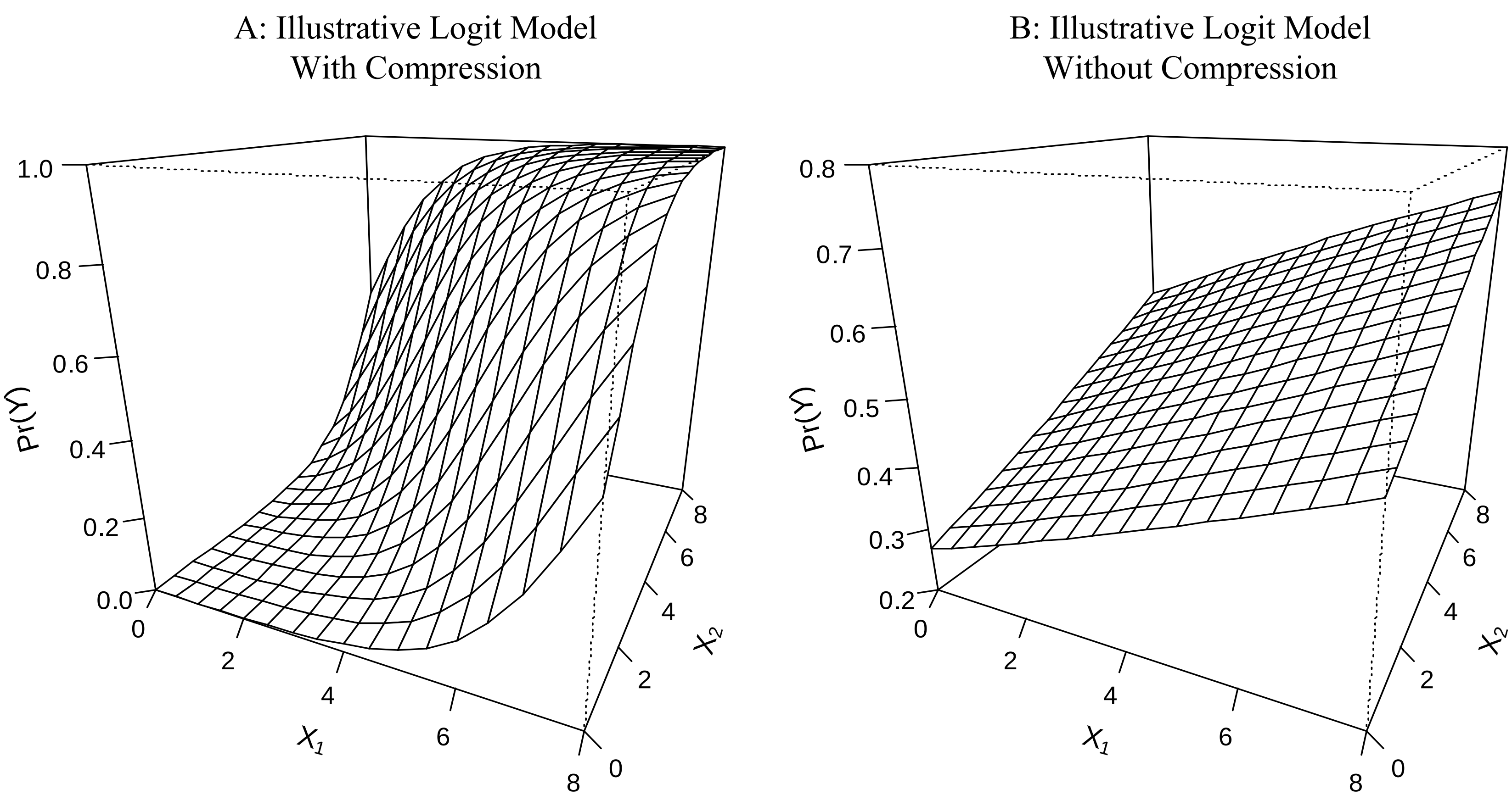


Figure 2: Logit Models Illustrating How Marginal Effects on Pr(Y) Vary with the Values of Independent Variables



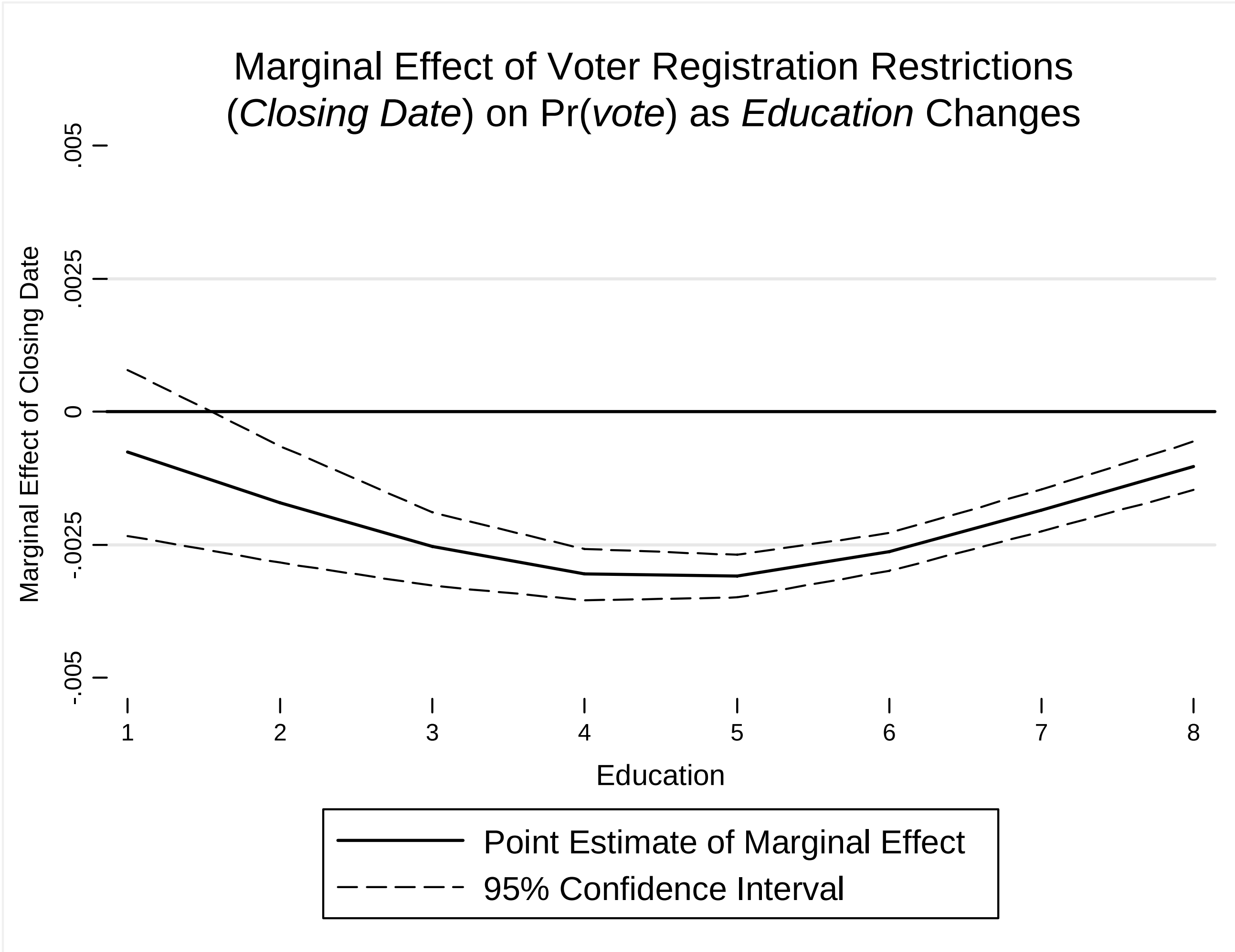
*The plot on the left illustrates the model Pr(Y) = $G\,(-4 + X_1 + X_2)$. The plot on the right illustrates the model Pr(Y) = $G\,(-4 + X_1 + X_2 - 0.30X_1X_2)$. In both cases, $G()$ is the logit link function. Arrows indicate the marginal effect (m.e.) of X_1 on Pr(Y), i.e., $\partial\Pr(Y)/\partial X_1$, at the point indicated.

Figure 3: Logit Models With and Without Appreciable Compression



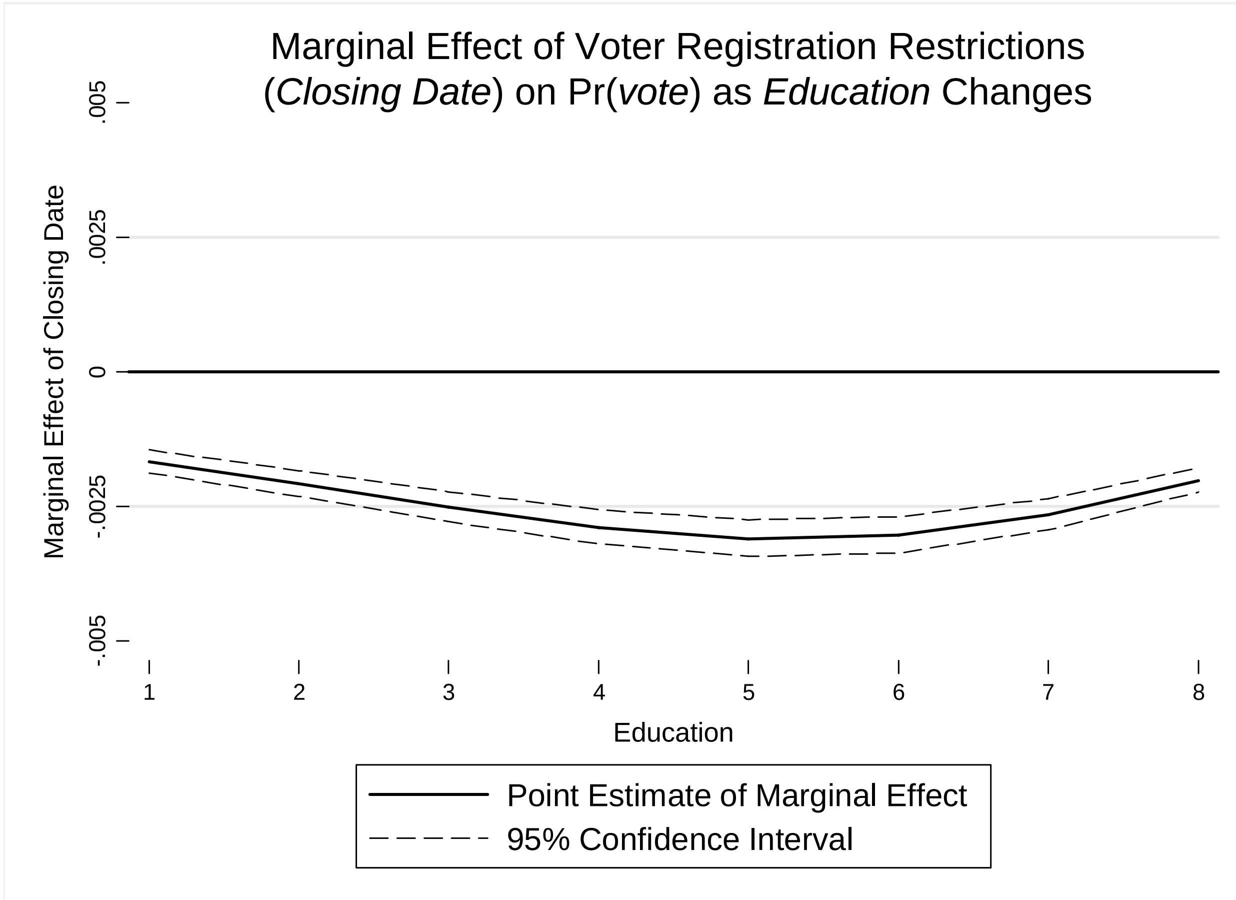
*The plot on the left illustrates the model Pr(Y) = G (– 8 + X_1 + X_2), and the plot on the right illustrates the model Pr(Y) = G (–1 + 0.125X_1 + 0.125X_2), both for DGPs in which X_1 and X_2 are confined to the [0,8] range. In both cases, G() is the logit link function.

Figure 4: Interaction Between Voting Registration Restrictions and Education in a Probit Model of Voter Turnout With Product Terms Included*



*Results correspond to the probit model of voter turnout depicted in Column 1 of Table 1. Marginal effects are computed with all continuous independent variables at their mean and all non-continuous variables at their mode. Marginal effects are calculated using Brambor, Clark and Golder's Stata code (available at http://homepages.nyu.edu/%7emrg217/interaction.html).

Unpublished Figure: Interaction Between Voting Registration Restrictions and Education in a Probit Model of Voter Turnout Without Product Terms*



*Results correspond to the probit model of voter turnout depicted in Column 2 of Table 1. Marginal effects are computed with all continuous independent variables at their mean and all non-continuous variables at their mode. Marginal effects are calculated using a modified version of Brambor, Clark and Golder's Stata code (available at http://homepages.nyu.edu/%7emrg217/interaction.html).